# Physics Experiments

# 2018

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# Preface

This book is intended for use as a textbook for physics experiments as a basic subject for the science-course students at Kyoto University. Physics experiment classes have the following three objectives. The first is for you to gain a deeper understanding of physics through experiments. Physics has advanced through experimental investigations of nature coupled with theoretical research. The study of physics, however, somehow tends to become desk learning through classroom lecture or textbooks. We would like you to have direct contact with physical phenomena through experiments to gain a deeper understanding of the structures and laws of nature. It is also valuable for you to experience for yourselves some of the important experiments that were performed by our predecessors. The second objective is for you to learn experimental techniques and methods of measuring physical quantities. Currently, various kinds of measuring devices are widely used in both natural and applied sciences. Therefore, no matter what field you are studying, you should become familiar with physical measuring devices and master the skills for correctly processing and appropriately evaluating the data that you obtain. The third objective is for you to learn how to write reports. Skills for distilling the essential points of what you have done and communicating them simply to other people are important not only for accurately and precisely reporting information but also for stimulating self motivation. You will learn how to format reports about the natural sciences by writing reports about your experiments.

In this revised edition, we have reexamined the subjects to be treated in the experiment class. And we have tried to make the description of the text easier to understand by providing the reader with the purpose and the background as well as the physical meaning of the experiment. Part 1 (the basics of experiments) of this book explains (I) what you should keep in mind to take the course, (II) measurement and the measuring device, (III) errors and their estimation, (IV) the graphs and the data processing, (V) how to write reports. Students should at least read these sections carefully and understand them before starting the first experiment. Part 2 (the actual experiments) explains the topics and contents of the experiments. For each subject, there will be the description about the purpose, the overview and the principle of the experiment. The instruments as well as the methods of the experiment will be then explained together with data analysis method and the related problems to be considered. The guidance given on the day of the experiment will presume that you have completed your preparations concerning the contents of the experiment. If you do not prepare, you will not understand what to do, and since you will beperforming the experiment by simply following the instructions in the textbook, the experiment will not go well and you will not learn enough from carrying it out. We plan to continue the revision of the textbook, the improvement of the content of the subject as well as the introducing the new topics of the experiment in the near future.

It has been pointed out for a long time that enough class hours for the experiments and the practices are not ensured at junior as well as senior high schools. Since the Physics Experiments provides the students with the opportunity in which they can participate in the class actively using their hands and brains, we expect their highly motivated learning. We hope you to enjoy the fun of experiments which cannot be obtained from the lectures, and strongly recommend you to have a chance of having a dialogue directly with the nature.

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# I. Introduction

## §1 Significance of Experiments

Physics is characterized, as in other natural sciences, by the process of analyzing the observations and measurements of the natural phenomena and set up and verify the hypothesis or the theory to explain the phenomena. Therefore, it is important to master basic concepts and fundamental technique by having the experience of experiments in addition to the theoretical study through attending the lectures and reading the textbook. The physics experiments to be treated in this textbook is one of the most fundamental subjects in common for the science-course students in parallel to the basic lecture courses to be mastered in physics. Here we ask you to note that it is not appropriate to regard the physics experiments as the learning to confirm what has been already learned in the theoretical studies. Actually, in the physics experiments, the first-year students will learn the subjects related to the quantum mechanics, before the most of them learn it at the lecture course of quantum mechanics. Namely, after they encounter the quantum phenomena through the course of physics experiments, they will learn the theory which explains the phenomena with deeper understanding.

## §2 What to learn

The apparatus and technology of the experiments in the field of reaserch and development has become more and more complicated and highly sophistcated and hence it takes much time and practice to master these technologies. No matter how complicated the device of the measurement is, however, the principle of the measurement is the building-up of the fundamental measurements.

Some of you may wish to go to the theoretical research area in the future and think that the experiments or practical training is not so necessary. In the theoretical research, in fact, the main task is to develop logical argument based on the mathematical expression and numerical computation, but it is still very important to have a contact with experiments. For example, you should contemplate what phenomena in nature appears as the result of the mathematical expression, and also what kind of physical quantity to be measured or what phenomena to be observed in order to establish theoretical expectation. It is an important ability for a theorist to draw concrete images from the mathematical expression. At the early stage to learn physics we recommend you to study theories as well as experiments actively without prejudice.

## §3 For your safety and ethics

It is important to study the content of the experiment in advance by reading the textbook. This is not only for doing experiments efficiently but also for carrying out the experiment safely. At the experiments, we sometimes heat up the material to high temperature (more than 1000K) or cool down the material to low temperature (77K). On the other hand in some experiment we use laser beam with a strong directionality which is dangerous for the eyes. In such experiments, a wrong handling of the experimental devices leads to an accident which injures not only one's self but also other people in the surroundings. You should keep it in mind to understand the contents of the experiments by reading the textbook prior to the class, and to carry out the experiment carefully following the instruction by the teacher. Taking out the insurance is recommended just in case.

In the actual experiment, one may have an experience of failure or mistake. You should not fear failures too much, and you may learn much from the failure. On the other hand the time of the experiment is limited and you should avoid the crucial mistake which requires performing the experiment again. In that sense, it is important to learn the content of the experiment in advance by reading the textbook. In particular, you should learn what the purpose is and what phenomena or what physical quantity to be observed/measured in advance.

What we would like to emphasize is that you should never falsify or fabricate the data even if you failed or made a mistake and if the measured data are substantially different from the expectation. Plagiarizing the other's report is strongly prohibited. These statements are coming not only from the ethical reason where you might lose the career of researcher or technician, but also from more positive reason in which the breakthrough is unexpectedly realized by overcoming the failure in a sincere effort.

## §4 What to keep in mind

Do not repeat measurement in a routine way just following the textbook. On the contrary, do the experiment with thinking about what physical quantity we are varying and what we are measuring. In doing so we have a consciousness of dialoguing with nature.

Although the experiment is carried out in pairs, do not heavily rely on your partner. You should work actively and collaborate with your partner in the measurement. We recommend you to discuss with each other. But the report should be written separately. It may be tedious to prepare the report, but that procees is an important part of the experiment. Even if you get a set of nice data, it is of no use unless you organize and examine well the results in your report. It is important to have a prediction for the results based on the physical consideration, but the conjecture due to prejudice and random guess should be avoided. Since the report is supposed to be read by other people, it must be written clearly and carefully. It is desiable to have efforts so that the the report conveys what the reporter thinks about to the readers.

Please prepare one notebook especially for the experiment. In this so-called lab notebook, you should record, in detail, the condition of the experiment, experimental data, calculational results as well as anything you have noticed during the experiment. These will be useful for the examination of the obtained data and also necessary for wiriting the report.

In the first time for the class, as an orientaion, we give a lecture in which we explain the purpose of the physics experiment, contents of learning, procedure how to carry out the experiment, how to write reports, and the schedule of the course. The dates of the orientation will be announced in advance (in the beginning of April for the 1st semester, at the end of September for the 2nd semester), so please check the announcement.

# II. Measurement and Measuring Devices

## §1 Basic Measuring Operations

To measure distance, for example, we will use a tape measure. We can find the distance according to the scale that was printed on the tape measure. In contrast to this kind of "analog" measurement according to a scale indication, "digital" measuring devices, which display numbers like on modern digital watches, have started to be used more often recently. In addition, there have been the experimental subjects using automatic measurement by PC. However, in this experimental physics course, we will first learn how to use basic measuring devices using graduated scales.

**a) Eliminate parallax** We will use a metal ruler, for example, to measure the length and width of your textbook. Place the ruler against the textbook and read the scale corresponding to its edge. However, you cannot place the ruler on the book as in Figure II.1 (a). You must stand the ruler up as in Figure II.1 (b). The reason is that in Figure (a), the scale and object are separated, and the corresponding scale position will be offset according to the viewing direction. Error will occur as a result. This is called parallax. In Figure (b), there is no parallax since the scale and target object are touching.

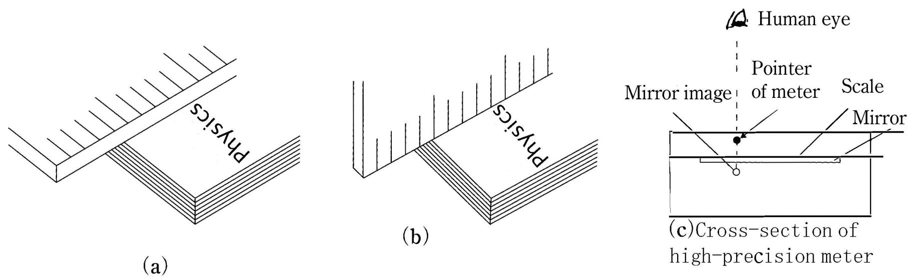



**Fig II.1** Measurement With a Ruler and High-Precision Meter Reading

As described above, we must always try to eliminate parallax to prevent excess error from occurring when reading a scale. For example, with a type of measuring instrument that indicates a value by moving a pointer over a scale printed on a dial like an electric meter, the scale and pointer cannot be made to touch as in Figure II.1 (b). Therefore to eliminate parallax, a mirror is placed right next to the scale and the measurement is taken by aligning the pointer and the image of the pointer reflected in the mirror. By doing this, the viewing direction is always perpendicular to the scale as in Figure II.1 (c). Therefore, the direction is fixed and error caused by parallax can be prevented.

**b) Prevent parallax in a telescope (microscope)** When using a telescope (microscope) for measurements, crosshairs are used as a reference point in the field of vision. In Figure II.2, if the image I of the object generated by objective lens O is not at the same position as crosshairs C when you look at it using eyepiece E, then image I and crosshairs C will be mutually offset according to the position of your eye and parallax will occur causing error. To eliminate this parallax, adjust the distance between C and E so that the crosshairs can be seen most clearly and then adjust the focus of the objective lens O (that is, the distance between O and C) so that the position of image I matches the position of crosshairs C. Try moving your eye forward and back and left and right. If the target object that you are viewing moves relative to the crosshairs, the positions of I and C do not match (that is, there is parallax). Therefore, to eliminate parallax, you should adjust the position of objective lens O so that there is no relative movement between the target object and crosshairs even if your eye moves.

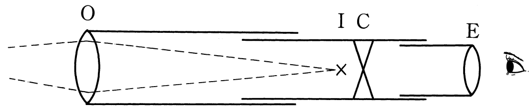



**Fig II.2** Parallax in a Telescope

**c) How to use a vernier scale** A vernier scale is used in various measuring devices since it enables a fraction (for example 1/10 or 1/20) of the smallest graduation of the scale to be accurately read mechanically. It is used, for example, in calipers and spectrometers

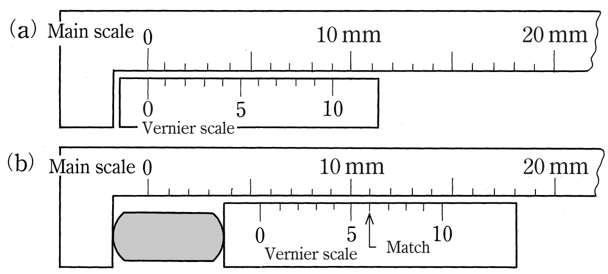



**Fig II.3** Vernier Scale

As shown in Figure II.3 (a), the vernier scale has graduations that divide 9 times the smallest graduation of the main scale (1 mm in the figure) into 10 equal parts so that 1/10 of the smallest graduation of the main scale (that is, 0.1 mm) can be read. When measuring the length of an object as in Figure II.3 (b), you first use the main scale to read the position of the 0 scale line of the vernier scale. Then, since only one of the graduation lines of the vernier scale will match a graduation line of the main scale, you read that vernier scale graduation. In Figure II.3 (b), since you first use the main scale to read 5 mm and then see that 6 on the vernier scale matches a graduation line of the main scale, the length is 5.6 mm. The reason for this is as follows. Since 9 graduations of the main scale (9 mm) have been divided into 10 equal parts (0.9 mm) on the vernier scale, the 1 graduation line of the main scale differs from the 1 graduation line of the vernier scale by 1/10 of the smallest graduation of the main scale (that is, 0.1 mm). As you traverse the vernier scale from 0, 1, 2, ⋯, the distance in terms of the main scale graduations is reduced 1/10 (0.1 mm) at a time, and the vernier scale graduation where the main scale and veneer scale graduation lines match represents the amount of discrepancy between the 0 graduation line of the vernier scale and the main scale graduation (5 mm in the figure). Generally, a vernier scale that divides (n - 1) times the smallest graduation of the main scale into n equal parts enables you to read up to 1/n of the smallest graduation of the main scale. Note that in a vernier scale for angular graduations, the smallest graduation of the main scale is 1/4 degree, and 29 times that value is divided into 30 equal parts. This enables you to read up to 30" of the angle (= 0.25 ° /30 = 1/120 ° ).

**d) Zero point correction or zero point adjustment** When input is zero, a measuring device may not indicate zero even though it should display zero. For example, in Figure II.3(a), the zero mark lines of the main and vernier scales may or may not match. If they do not match, the amount they differ by must be appropriately added to or subtracted from the measured value. This is called zero point correction.

Also, some measuring devices like an electric meter enable you to adjust the pointer to zero when it is offset from zero. This is called zero point adjustment. Zero point adjustment must be done for every measurement in a device such as a mercury barometer

**e) Verification tolerance** Many of the various types of measuring devices that are used in physics experiments are not just used in the realm of physics but are also widely used in general technical and engineering fields. As a result, the accuracy of those measuring devices must be guaranteed throughout our society. Therefore, allowable error has been determined by laws and regulations such as the Measurement Law, and these devices are verified in advance in our marketplaces. This allowable error is called **tolerance**. This is described specifically in the sections for individual measuring devices.

Note that meters and gauges normally change with the passage of time, and it is normal for the precision to worsen as they break and are repaired. Therefore, if absolute values are strictly required, you must have someone such as the manufacturer of the meter or gauge certify the error of that device in advance.

**f) Measuring devices with digital display** The measuring devices with digital display are convenient since everyone can read the same value without suffering from the problem of parallax. In the case of digital multimeter (DMM), for example, the measure valus are displayed as 3 to 6 digits. The question is how many digits we should take as the measured values. In the case where the displayed value is stable we take upto the minimum digit. If the displayed value is unstable we take the upto the maximum digit which fluctuates.

## §2 Basic Measuring Devices

**a) Caliper** As shown in Figure II.4, the caliper is equipped with jaw AB at one end of the main scale and jaw CD and a vernier scale, which slide along the main scale. BD is used to measure the outer diameter of a cylinder, for example, AC is used to measure the inner diameter, and E is used to measure the depth of a groove. In any case, you slide the caliper until it lightly touches the object to be measured, and you must not press on the object too strongly. The screw head attached to the vernier scale is for fixing the position of this scale. Be sure to loosen this screw before sliding the scale. Be careful since if you forcibly slide the scale with this screw tightened, you will damage the body so that it will no longer slide smoothly.

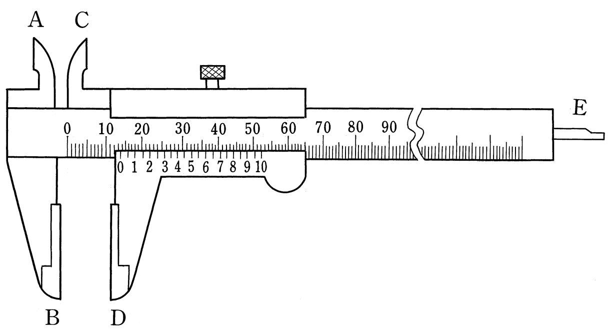



**Fig II.4** Caliper

**b) Micrometer** As shown in Figure II.5, anvil A is affixed to one end of curved frame F, and sleeve B is affixed to the other end. A 1/2 mm scale is engraved on the outside of B, a female screw with a 1/2 mm pitch is engraved on its inner surface, and a screw spindle whose leading edge is cut perpendicular to its axis is inserted inside this sleeve. Since thimble T is affixed to C, when T makes one revolution, C advances or retracts 1/2 mm. A scale that divides 1 circumference into 50 equal parts is engraved on one end of T. Therefore, the graduations of T correspond to 1/100 mm. When C is touching A, the end of T is aligned with the zero of the scale of B and the mark line on B is aligned with the zero of the scale of T. Ratchet R is attached on the end of T to keep the force that is applied fixed when a object is interposed between A and C for taking a measurement. If the force applied on C exceeds a fixed value when R rotates, the ratchet will begin to slip. Therefore, when taking a measurement, rotate T until just before the object is held between A and C and then use R to advance or retract C. D is a clamp that is close when turned to the left. Never rotate T when this clamp is closed. Verify the zero point and correct it if necessary. You should adjust the zero point by rotating B using the calibration rod that is included. Note that the ends of A and C should be left opened when the micrometer is not in use.

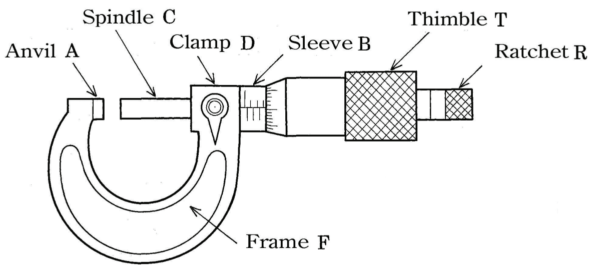



**Fig II.5** Micrometer

**c) Mercury barometer (Fortin barometer)** If a glass tube 1 meter long is closed at one end, filled with mercury, and then the open end is inserted vertically into a basin of mercury, the column of mercury will descend to the height that balances the atmospheric pressure, where it will remain stationary. A vacuum in which mercury vapor remains (approximately $1 \times 10^{-3}$mmHg at room temperature) will occur at the top of this Torricelli mercury column. The atmospheric pressure is obtained from the height of this mercury column.

In Figure II.6, A is a glass mercury container, glass tube B, which is closed at the top, is affixed to A, and a Torricelli mercury column is created inside of B. A is exposed to the open air, the base of A consists of leather bag C, and the position of the surface of the mercury inside A can be raised or lowered by screw D.

To take a measurement, first fix screw E while the barometer is freely suspended vertically, and then move screw D to adjust the surface of the mercury inside A so that it reaches the top of the ivory needle N affixed to A. The tip of needle N is a reference, a scale is engraved on the metal protective tube G, and a vernier scale V is moved by screw S. Although the surface of the mercury is curved concavely upward by surface tension, you measure the height at its zenith. There are two types of scales on the left and right. These are the pressure units mmHG, which measures the height of the column of mercury in millimeters (mm), and millibars (mb). The pressure unit in the SI international system of units is $1\text{N/m}^2 = 1$ Pa (pascal), and hPa (hectopascals) began to be used for atmospheric pressure in December 1992. Conversion relationships are as follows.

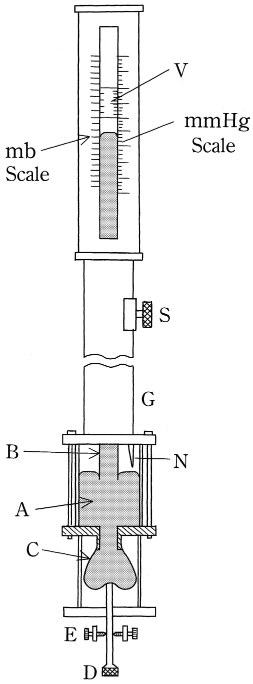



**Fig II.6**

$$1\,\text{b(bar)} = 10^5\,\text{Pa}, \quad 1\,\text{mb} = 1\,\text{hPa}$$

$$1\,\text{mmHg} = 1\,\text{Torr} = (101325/760)\,\text{Pa}$$

and, 1 (standard) atmosphere (atm) $= 760\,\text{mmHg} = 1013.3\,\text{mb} = 1013.3\,\text{hPa}$

The following equation can be used for corrections to a barometer reading. If p denotes the pressure corresponding to the height h of the mercury column, g denotes gravitational acceleration, and ρ denotes density, then the following relationship holds.

$$p = \rho g h$$

Therefore, the indicated value $h$ of the barometer for a certain pressure $p$ varies according to the values of ρ and $g$. Since the density of mercury is 13.5951 g/cm$^3$ at 0 ° C and 13.5214 g/cm$^3$ at 30 ° C, a difference of approximately 0.5% occurs in the indicated value, which cannot be ignored. On the other hand, for gravitational acceleration, since the standard value is 9.80665 m/s$^2$ and the value at Kyoto is 9.79708 m/s$^2$, this difference is approximately 0.1%. In addition, there is also an effect from the surface tension of mercury, but this is normally corrected.

**d) Psychrometer (wet-and-dry bulb thermometer)** If $p$ denotes the partial pressure of water vapor in the air and $p_s$ denotes the saturated vapor pressure of water at that temperature, then the (relative) humidity $H$ is defined by the following formula.

$$H = (p/p_s) \times 100\ (\%)$$

If two thermometers are placed next to each other and one is wrapped in a thin, wet cloth, the temperature will drop due to evaporation of the water. Since the evaporation of water also varies due to the level of humidity in addition to the room temperature, the humidity is obtained from this drop in temperature. The relationship between these values is shown in Appendix 13 (page 142) and displayed on the psychrometer.

**e) Needle type electric meter** There are various models depending on the operating principles.

① **Moving coil type** This is most widely used for direct current. If current flows in a coil that can rotate in a magnetic field created by a permanent magnet, a coupling of forces is generated by the electromagnetic forces. The needle moves until these forces are balanced by the elastic force due to the fishing line (for precision) or coil.

② **Moving iron vane type** This is used for direct and alternating current. It uses the property that an iron vane is attracted or repelled if current flows in a coil. It is not very precise.

③ **Electrodynamometer type** This is used for direct and alternating current. It uses an electromagnet due to the measured current in place of the permanent magnet of the moving coil type. It can also measure electric power.

④ **Rectifier type** This is used for alternating current. It rectifies alternating current to convert it to direct current, which is measured by a moving coil-type electric meter. Note that a rectifier may change with the passage of time.

Besides the various symbols shown above, which appear on the scale face, one of the following 5 classes (class 0.2, class 0.5, class 1.0, class 1.5, and class 2.5) will also appear depending on the allowable error (tolerance). This class value represents that the meter error is within the class value (%) of the full scale (for example, with a 0.2 class milliampere meter with a fullscale value of 100 mA, the tolerance is $100\,\mathrm{mA} \times 0.2\% = 0.2\,\mathrm{mA}$). In addition, the scale face will also contain symbols such as (A) indicating an ammeter, (V) indicating a voltmeter, (—) indicating direct current, (∼) indicating alternating current, (⊥) indicating that it should be used standing vertically, ( ) indicating that it should be used horizontally, as well as the year of manufacture, etc.

An ammeter is entered in series with the circuit to be measured as shown in Figure II.7 (a), but its input impedance $Z_i$ must be much smaller than the circuit impedance $Z$ (as indicated by $Z_i \ll Z$). On the other hand, a voltmeter, in contrast, is entered in parallel with the circuit to be measured as shown in Figure II.7 (b). Its input impedance $Z_i$ must be related to $Z$ as $Z_i \gg Z$. For a complex circuit, be sure not to make a mistake when wiring the ammeter or voltmeter.

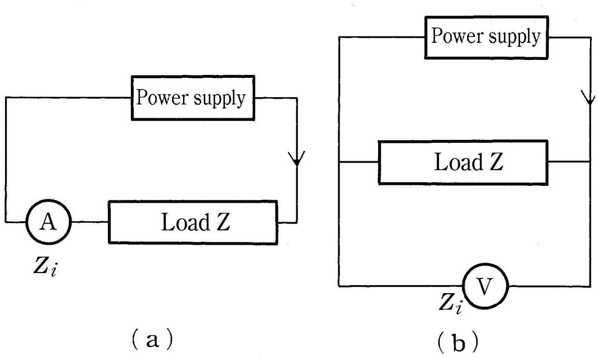



**Fig II.7** Ammeter or Voltmeter Measurement

(Note) For a current measurement, a known resistance is often entered in series with the load $Z$ and the voltages generated at both sides of it are measured.

# III. Error and Error Handling

## §1 Measurement and Error

When you associate a length with the graduation of a ruler or tape measure or associate a weight with the graduation pointed to by the needle of a scale, you establish a quantitative standard and determine what multiple of that standard the relevant graduation is proportionate to. Measurement is the task of obtaining a quantitative result in this way by comparing it with a standard, and a measured value is the value that is obtained.

The difference between the **measured value** and **true value** is defined as **error**:

$$\text{Error} = \text{Measure value} - \text{True value} \qquad \text{(III.1)}$$

The quantity that we can find, however, is only a measured value and we cannot know what the true value is. Therefore, it is now recommended as the international standard, to use the word "**uncertainty**"which represents fluctuations of the measure values. In this text, following the common usage in physics, the word "error "is used to mean "uncertainty".

**a) Class of Errors** Errors can be categorized ino two classes: systematic errors and sccidental errors.

1. **Systematic error**

This is an artificial error due to a defect in the measuring device or a blunder by or inexperience of the person taking the measurement (since this affects the measured value in a fixed way, it is called systematic error)

2. **Accidental error**

This is an error that tends to occur randomly (called accidental error), which cannot be controlled even by a skilled person. Although every effort must be made to eliminate systematic error as much as possible, since random error should appear as positive and negative error values in almost the same proportions due to its nature, its effect can be minimized by increasing the number of times the measurements are taken.

Now the question is how to obtain most probable value from the measurement. As we will see below, in order to get the most probable value, we have to repeat the measurement as many as possible and take avarage value.

**b) most probable value** If we perform the measurement for $n$ times and get the values $X_1$, $X_2$, $\cdots$, $X_n$, then we have the average value (mean value) $\bar{X}$ given as

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{III.2}$$

When we increase the number of times for the measurement $n$, we approach the true value $X_0$. We define the error for each measurement $x_i$ as

$$x_i = X_i - X_0 \tag{III.3}$$

while the quantity called **deviation** is defined as the difference between the measured value and the mean value $\bar{X}$

$$\text{deviation} = \text{measured value} - \text{mean value} \tag{III.4}$$

Thus summing up the equation (III.3) with respect to $i$ from 1 to $n$ we get

$$\sum_{i=1}^{n} x_i = n(\bar{X} - X_0) \quad \text{or} \quad \bar{X} = X_0 + \frac{1}{n}\sum_{i=1}^{n} x_i \tag{III.5}$$

If we take the limit $n \to \infty$, then $\frac{1}{n}\sum_{i=1}^{n} x_i \to 0$, and hence $\bar{X}$ approaches $X_0$. Namely, as the number of the direct measurement $n$ becomes larger, the average value becomes more reliable.

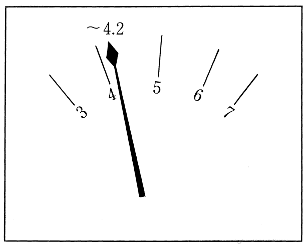



**Fig III.1** Scale Reading of an Analog Instrument (Reading Up to 1 10 of the Graduation)

**c) Measurement precision** When obtaining a measured value by reading the graduations of a scale, you can read up to 1/10 of the smallest graduation by eye (see Figure III.1). However, since this reading has an uncertainty of at least $\pm\frac{1}{10}$, the final digit of the measured value contains an error of at least 1/10 of 1 graduation even if nothing in the equipment settings causes error. If the measured value is written as 23.4 g, it indicates that an error of ±0.1 g can exist. Therefore, when writing the measured value, since the degree of error differs by 1 digit between 23.4 g and 23.40 g, the final digit 0 cannot be omitted. Generally, the part that can be trusted among the digits used to indicate a measured value is called the significant digits. If you want to clearly indicate the significant digits part, write $2.0 \times 10^{-2}$ cm or $1.230 \times 10^{3}$ K rather than 0.020 cm or 1230 K.

Since the precision of a measurement is also related to the size of the measured quantity itself rather than just to the absolute size of the error, a concept called relative error is used.

$$\text{Relative error} = \frac{\text{Error}}{\text{True value or most probable value}} \tag{III.6}$$

A 10 cm error in a measurement of approximately 100 m and a 0.01 mm error in a measurement of approximately 1 cm have the same relative error of $10^{-3}$.

## §2 Error Evaluation

**a) Three axioms of error** If we repeat the measurement of a certain quantity, everytime we get a different measured value due to accidental errors. Now we take the deviation of the measured value to the horizontal axis, and divide it into small intervals of size $\varepsilon$, and let's assume the frequency for the measured value to fall in the $i$-th interval given by $n_i$. Then we have the frequency of the deviation shows the distribution illustrated in the following figure.

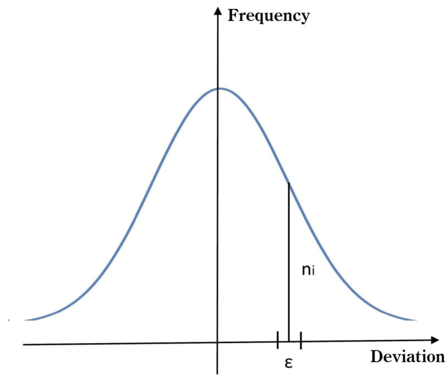


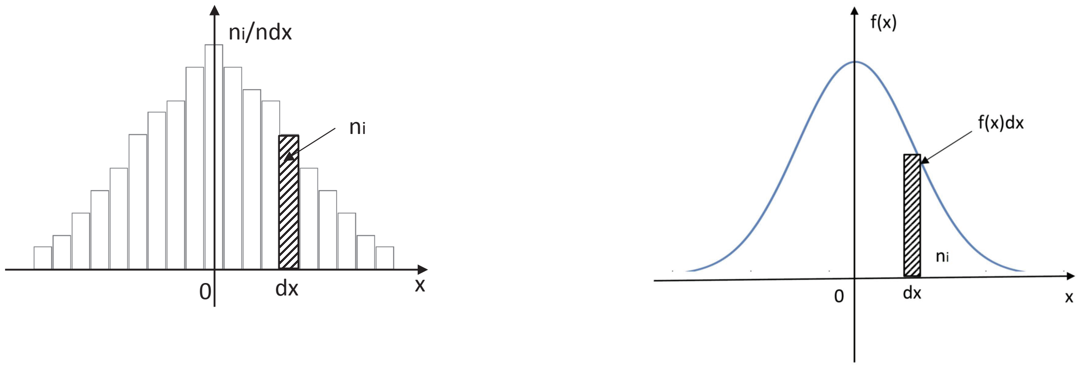

**Fig III.2** Frequency Distribution

When $n$ is small the curve of the distribution looks irregular with respect to the height, but for the case that $n$ is large enough, the curve becomes left-right symmetric, high in the center and smoothly damping at both ends. We call this curve a **frequency distribution**. In the case when the number of times for the measurement extremely large, we can regard the deviation as the error, the following **three axioms of error** holds.

1. Positive and negative errors with equal absolute values occur equally.

2. Errors with small absolute values occur more often than errors with large absolute values.

3. Errors with absolute values greater than or equal to a certain amount do not substan-




(a) (b)

**Fig III.3** (a) discrete distribution (b) probability function

tially occur.

**b) Probability distribution and probability function** As we mentioned above, if the number of times for measurement is sufficiently large, the deviation can be regarded as the error. Suppose that the frequency for the error to take values between $x$ and $x + dx$ is given by $n_i$ and the total number of measurements is $n$, then the probability for the error to exist between $x$ and $x + dx$ is equal to $n_i/n$. This is equal to the area of the elongated rectangle with width $dx$ shown in Figure III.3 (a). Now in the limit of $dx$, which is the width of the rectangle with height $n_i/ndx$, going to the infinitesimally small, we get the smooth curve shown in Figure III.3 (b). If we denote the curve by the function $f(x)$, the probability for the error to take a value in the interval $x$ and $x + dx$ is given by $f(x)dx$. We call $f(x)$ as **probability function**. When integrated over entire interval: from $-\infty$ to $\infty$, we get

$$\int_{-\infty}^{\infty} f(x)dx = 1, \tag{III.7}$$

which is the function derived by C. F. Gauss and is called **normal distribution** ( or **Gauss distribution**) and is given as (See the Appendix 1 § 1 Probability and Statistics)

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{x^2}{2\sigma^2}\right] \tag{III.8}$$

where $\sigma$ is a quantity called "standard deviation"and is a constant related to the accuracy of the measurement.

Now let us derive the above normal distribution. Suppose that the true value is $X_0$ and the measured values for $n$ times are $X_1,\ X_2,\ \cdots,\ X_n$, then each error reads

$$X_i - X_0 = x_i \qquad (i = 1, \cdots, n) \tag{III.9}$$

The probability for the error to take a value in the infinitesimal interval $x_i - \varepsilon$ と $x_i + \varepsilon$ is found to be

$$f(x_i) \cdot 2\varepsilon \tag{III.10}$$

The probability $P$ for these errors to occur simultaneously is obtained by using the law of probability for a composite event.

$$P = f(x_1)f(x_2)\cdots f(x_n)(2\varepsilon)^n \tag{III.11}$$

In the limit of $n$ going to $\infty$, the average value ( mean value) $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ becomes the most probable value. Thus if we regard $X_0$ as $\bar{X}$, $P$ becomes maximum. We find out the functional form of $f(x)$ from this condition. Namely, in order to get the maximum value of $P$, we take the logarithm of $P$ and set the derivative of it with respect to $X_0$ to be zero.

$$\frac{d}{dX_0}\log P = \frac{f'(x_1)}{f(x_1)}\frac{dx_1}{dX_0} + \cdots + \frac{f'(x_n)}{f(x_n)}\frac{dx_n}{dX_0} = 0 \tag{III.12}$$

By differentiating both sides of Equation (III.9) with respect to $X_0$ we have,

$$\frac{dx_1}{dX_0} = \cdots = \frac{dx_n}{dX_0} = -1 \tag{III.13}$$

and setting $f'(x)/f(x) \equiv g(x)$ we get

$$\sum_{i=1}^{n} g(x_i) = 0 \tag{III.14}$$

From the axiom of error, when $n$ is large enough, positive and negative errors with equal absolute values occur equally, hence the total sum of errors vanish.

$$x_1 + \cdots + x_n = \sum_{i=1}^{n} x_i = 0 \tag{III.15}$$

Since the $n$-th error can be rewritten as

$$x_n = -(x_1 + x_2 + \cdots + x_{n-1}) \tag{III.16}$$

we can regard $x_n$ as the dependent variable of the remaining errors $x_1, x_2, \cdots, x_{n-1}$. Hence differentiating (III.14) by $x_1$, we find

$$g'(x_1) + g'(x_n)\frac{\partial x_n}{\partial x_1} = 0 \tag{III.17}$$

Now noting that $\partial x_n/\partial x_1 = -1$ we get

$$g'(x_1) = g'(x_n) \tag{III.18}$$

and similarly we have

$$g'(x_1) = g'(x_2) = \cdots = g'(x_n) \tag{III.19}$$

Thus $g'(x)$ turns out to be a constant. Writing this constant as $a$ we obtain

$$g(x) = ax + b \tag{III.20}$$

where $b$ is an integration constant. Substituting the above equation into (III.14) and using (III.15) we find $b = 0$ and finally get

$$g(x) = \frac{f'(x)}{f(x)} = ax \tag{III.21}$$

Therefore integrating (III.21) over $x$ we obtain the following functional form of $f(x)$:

$$f(x) = C\exp(\frac{a}{2}x^2) \tag{III.22}$$

where $C$ is a constant. Note that in the limit $x \to \infty$, we have to have $f(x) \to 0$, This is satisfied only if $a$ is a negative number. Thus putting $a/2 = -h^2$ we get

$$f(x) = C\exp(-h^2x^2) \tag{III.23}$$

which can be normalized by taking

$$C = \frac{h}{\sqrt{\pi}} \tag{III.24}$$

Rewriting $h = 1/(\sqrt{2}\sigma)$, the desirable form of $f(x)$ can be finally derived

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{III.25}$$

**c) Mean value and standard deviation** Let us consider the normal distribution in which the mean value (most probable value) is $X_0$:

$$f(X - X_0) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(X - X_0)^2}{2\sigma^2}\right] \tag{III.26}$$

When we get the measured values $X_i$ $(i = 1, 2, \cdots, n)$ after we have done $n$ times measurements, **root mean square** or **standard deviation** is defined as

$$S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - X_0)^2} \tag{III.27}$$

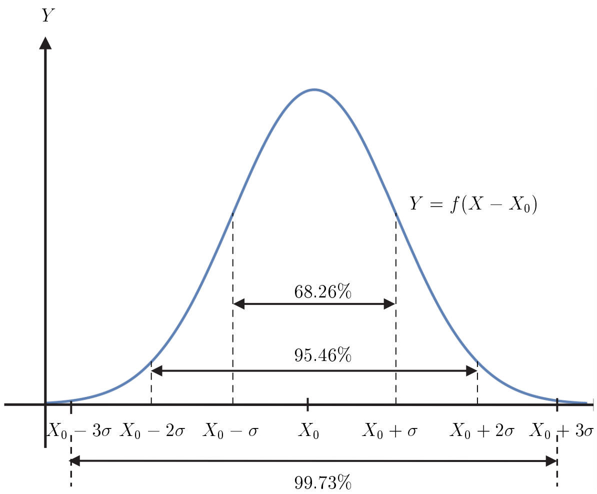
The square of this standard deviation is called **variance** which we denote by $S^2$ as follows,

$$S^2 = \overline{(X - X_0)^2} = \frac{1}{n}\sum_{i=1}^{n}(X_i - X_0)^2 \tag{III.28}$$

Computing the variance $S^2$ for the above normal distribution we get

$$S^2 = \int_{-\infty}^{\infty}(X - X_0)^2 f(X - X_0)dX = \int_{-\infty}^{\infty}x^2 f(x)dx = \sigma^2 \tag{III.29}$$

Thus the standard deviation $S$ is equal to $\sigma$. Hence the parameter which represents the width of the probability curve corresponds to $\sigma$. The probability curve $f(x)$ shows the inflection point at $x = X_0 \pm \sigma$. The integration of $f(x)$ over $x$ for the interval $[X_0 - \sigma, X_0 + \sigma]$ is about 0.68. Therefore for about 70% of the measured values, the absolute value of the error is smaller than $\sigma$. We can use $\sigma$ as the indication of the reliability for the measured value. We have shown in Figure (a) and Table (b) of III.4 the values of integral for the intervals $[X_0 - 2\sigma, X_0 + 2\sigma]$ and $[X_0 - 3\sigma, X_0 + 3\sigma]$ in units of percent.




(a)

| Interval | Probability |
|---|---|
| $[X_0 - \sigma, X_0 + \sigma]$ | 68.26% |
| $[X_0 - 2\sigma, X_0 + 2\sigma]$ | 95.46% |
| $[X_0 - 3\sigma, X_0 + 3\sigma]$ | 99.73% |
| $[X_0 - 4\sigma, X_0 + 4\sigma]$ | 99.99% |

(b)

**Fig III.4** (a) normal distribution (b)interval of normal distribution and probability

In our physics experiment, however, we take a view that the true value $X_0$ is an unkown quantity. Therefore we adopt the mean value $\bar{X}$ as the most probable value instead of the true value. (See the Appendix 1- § 2 the Method of Maximum Likelihood)

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{N} X_i. \tag{III.30}$$

Here we note that the standard distribution in this case is not (III.27). As we will discuss, by using the following quantity

$$\sigma_{\exp} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2} \tag{III.31}$$

we express the result of the measurement as

$$\bar{X} \pm \sigma_{\exp} \tag{III.32}$$

The reason why the denominator now becomes $n - 1$ is the following. Since we have once used the measured data to determine $\bar{X}$, the number of independent data reduced to $n - 1$.

If we define the error $\delta_i$ as

$$\delta_i \equiv X_i - X_0 = X_i - \bar{X} + \bar{X} - X_0 \tag{III.33}$$

and the deviation $\Delta_i$ as

$$\Delta_i \equiv X_i - \bar{X} \tag{III.34}$$

then we have the relation $\delta_i = \Delta_i + \bar{X} - X_0$, $\sum_{i=1}^{n} \Delta_i = 0$ and hence we find

$$\sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} \Delta_i^2 + n(\bar{X} - X_0)^2 \tag{III.35}$$

where

$$(\bar{X} - X_0)^2 = \frac{1}{n^2}\left[\sum_i (X_i - X_0)\right]^2 = \frac{1}{n^2}(\sum_i \delta_i)^2 = \frac{1}{n^2}(\sum_i \delta_i^2 + \sum\sum\nolimits_{i \neq j} \delta_i \delta_j) \tag{III.36}$$

The second term in the parenthesis of the above equation, $\sum\sum_{i \neq j} \delta_i \delta_j$, does not increase with $n$. Therefore if $n$ is large enough, the following relation holds in a good approximation

$$(\bar{X} - X_0)^2 \simeq \frac{1}{n^2}\sum_i \delta_i^2 \tag{III.37}$$

and thus we get

$$\left(1 - \frac{1}{n}\right)\sum_i \delta_i^2 = \sum_i \Delta_i^2 \tag{III.38}$$

Namely we find the following relation:

$$\sigma^2 = \frac{1}{n^2}\sum_i \delta_i^2 = \frac{1}{n-1}\sum_i \Delta_i^2 \tag{III.39}$$

which leads to

$$\sigma_{\text{exp}} = \sqrt{\frac{\sum_i (X_i - \bar{X})^2}{n-1}} \tag{III.40}$$

**d) Law of propagation of errors** Let us now suppose a physical quantity $z$ which can be determined by the two independent physical quantities $x$, $y$, and the functional realtion is given by

$$z = f(x, y)\ . \tag{III.41}$$

If the errors for the measuremnet of $x$ and $y$ are assumed to be $\sigma_x$ and $\sigma_y$, respectively, then we get the error for $z$, $\sigma_z$ which arises from the propagation of errors of $x$ and $y$ and is given by the following equation:

$$\sigma_z = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 {\sigma_x}^2 + \left(\frac{\partial f}{\partial y}\right)^2 {\sigma_y}^2} \tag{III.42}$$

where we have introduced $\partial f/\partial x,\ \partial f/\partial y$ which are called the partial derivatives of $f(x, y)$ with respect to $x$ and $y$, respectively, and are defined as follows.

**Partial derivative** For a function $f(x, y)$ which depends on the two independent variables $x$ and $y$, we fix $y$ to a constant and vary $x$ by an infinitesimal quantity $\Delta x$. In this case if the following limit

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x, y)-f(x, y)}{\Delta x}=\frac{\partial f(x, y)}{\partial x} \tag{III.43}$$

exist, we call this limiting value the **partial derivative** of $f$ with respect to $x$. We also denote the right-hand side as $f_x(x, y)$. Similarly in the case we keep $x$ to a constant and vary $y$ by $\Delta y$, the limiting value:

$$\lim_{\Delta y\to 0}\frac{f(x, y+\Delta y)-f(x, y)}{\Delta y}=\frac{\partial f(x, y)}{\partial y} \tag{III.44}$$

is called the partial derivative of $f$ with respect to $y$, and we also denote it as $f_y(x, y)$. For example let us consider a function $f(x, y) = x^2y$, and take partial derivatives with respect to $x$ and $y$. Then we obtain the following results.

$$\frac{\partial f(x, y)}{\partial x}=2xy, \quad \frac{\partial f(x, y)}{\partial y}=x^2 \tag{III.45}$$

Now, in the previous case, the physical quantity $z$ depends only on two variables $x,\ y$. In more general case where the function $z$ depends on $n$ variables, $x_1,\ x_2,\ \cdots,\ x_n$ as given by

$$z = f(x_1, x_2, \cdots, x_n) \tag{III.46}$$

the error of $z$ turns out to be

$$\sigma_z=\sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2{\sigma_{x_1}}^2+\left(\frac{\partial f}{\partial x_2}\right)^2{\sigma_{x_2}}^2+\cdots+\left(\frac{\partial f}{\partial x_n}\right)^2{\sigma_{x_n}}^2} \tag{III.47}$$

This rule to determine the error is called **law of propagation of errors**.

**Example** For a spherical matter with diameter $D$ and mass $W$, the volume density $\rho$ can be expressed by $D$ and $W$ as

$$\rho=\frac{6W}{\pi D^3}\ . \tag{III.48}$$

If the errors of $D$ and $W$ are given by $\sigma_D$ and $\sigma_W$, respectively, evaluate $\sigma_\rho$, the error of $\rho$, applying the law of propagation of errors.

**Solution** Differentiating $\rho$ with respect to $D$ and $W$ we get

$$\frac{\partial\rho}{\partial D}=-\frac{18W}{\pi D^4}, \quad \frac{\partial\rho}{\partial W}=\frac{6}{\pi D^3}\ . \tag{III.49}$$

Hence approximating the true value by the mean value we have

$$\sigma_\rho=\sqrt{\left(\frac{18\bar{W}}{\pi\bar{D}^4}\sigma_D\right)^2+\left(\frac{6}{\pi\bar{D}^3}\sigma_W\right)^2} \tag{III.50}$$

which is nothing but the standard deviation of the density $\rho$.

**e) Standard deviation of mean value** In many cases, the true value is unknown for us, we try to obtain the standard deviation $\sigma$ from the mean value $\bar{X}$. The mean value is the most probable value, but it is accompanied by the error. In order to express the reliability of the mean value, we consider its standard deviation.

When we repeat the operation of taking the mean value $\bar{X}$ by performing the measurement for $n$ times, we get the distribution of $\bar{X}$. If we consider this process as doing the $n$ independent measurements, the mean value can be regarded as an indirect quantity to be determined by $n$ variables as

$$\bar{X} = f(X_1, X_2, \cdots, X_n) = \frac{1}{n}(X_1 + X_2 + \cdots + X_n) \tag{III.51}$$

Now note that these variables $X_1,\ X_2,\ \cdots,\ X_n$ possess errors $\sigma_{X_1},\ \sigma_{X_2},\ \cdots,\ \sigma_{X_n}$. So from the law of error propagation we find

$$\sigma_{\bar{X}} = \sqrt{\left(\frac{\partial f}{\partial X_1}\right)^2 \sigma_{X_1}^2 + \cdots + \left(\frac{\partial f}{\partial X_n}\right)^2 \sigma_{X_n}^2} = \sqrt{\frac{1}{n^2}\sigma_{X_1}^2 + \cdots + \frac{1}{n^2}\sigma_{X_n}^2} \tag{III.52}$$

In the case where all the errors are equal, $\sigma_{X_1} = \cdots = \sigma_{X_n} = \sigma_X$ we get

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} \tag{III.53}$$

On the other hand, since $\sigma_X$ is given by $\sqrt{\sum_i (X_i - \bar{X})^2/(n-1)}$, the standard deviation of the mean value turns out to be

$$\sigma_{\bar{X}} = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n(n-1)}}\ . \tag{III.54}$$

Therefore $\sigma_{\bar{X}}$ stands for the reliability of the mean value.

# IV. Graph and Data Analysis

## §1 Importance of Graph in the Experiment

Graphs are important not only for displaying the experimental results but also for performing the experiment itself. It is not uncommon to find out the relation between one physical quantity and another one only after we draw the graph. By drawing the graphs in the course of the experiment we can check if the obtained data are reasonable ones, namely, if we are free from breakdown of the devices as well as from the mistake in the measurement. We can also decide which is the most suitable point for the next measurement. For example, when the experimental data just obtained deviates from the trend of the previously obtained data, we should measure again the last data point. We should also make the measuring points more dense in the region where the data vary in a large scale. If we draw the graph after taking the whole data points, we cannot take advantages mentioned above. In that case, moreover, we might notice a mistake after the experiment is over and we would have to do the experiment again from the beginning and would have a dangerous risk for waisting time and labor as well as resources.

Since in our experiment of this class, for good or bad, we know the results of measurement to a certain extent in advance, or the data points to be measured are instructed, it is difficult for us to notice the importance of drawing graphs during the experiment. Think about your graduation work or starting the reseach after your graduation. You would clever enough to realize the importance of the above-mentioned points. We recommend you to draw the graphs during the experiment so that you can have a chance to exercise for the actual experimental research in the future.

## §2 Graph

Let's try to use graph paper as big as possible. We must consider the maximum and minimum values of the data to be plotted and select scale units so that the graph is not lopsided on the graph paper. The lower left corner need not be set as the coordinate origin (0, 0). Consider how to mark off the graduations on each of the vertical and horizontal axes and enter the physical quantities, units, and scales. A graph with units omitted is meaningless. Next, when plotting the data, enter marks such as circles with diameters of 1 or 2 mm (a suitable size for distinguishing them from the lines connecting two consecutive points) so that the data point positions are apparent. When the sequence of points are arranged practically along a straight line or a certain curve, you can simply connect them with a smooth line. However, since the points are often generally scattered, you should look over the entire set of data and draw a curve so that the offsets from the curve are in balance. Do not draw a broken line graph and do not assume some conclusion and draw a line to match it.

Since the graph paper that will be used should be rectangular, use the vertical and horizontal directions properly according to the data to be plotted and determine the scales so that the graph can be drawn almost in a diagonal direction. The three types of graph paper that will be used for the experiment reports are for millimeter graphs, log-log graphs, and semi-log graphs. When the plot is not a straight line on millimeter graph paper, you should be prepared to try plotting by assuming various functional relationships. One possible scheme is to try using log-log or semi-log graph paper. Since values are compressed if the logarithm is taken, this is convenient when the measured values are distributed over a wide range.

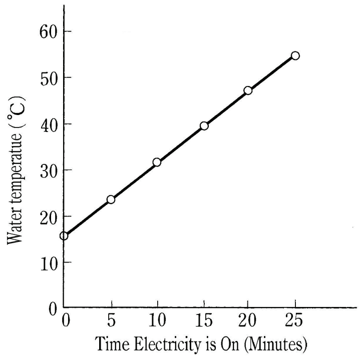



**Fig IV.1** Characteristics of an Electric Heater

Based on the preceeding theoretical or experimental research, we can expect the data points and plot the data on the graph paper as full as possibe. It may sometimes happen, however, that the appearance of the unexpected data cause the data points to run out of the preset scale of the graph. In such a case, we attach a new graph paper to the edge where the data run off and draw the data on it. We finally redraw the graph for the report.

## §3 Data Analysis

**a) Millimeter graph scale** When the sequence of data points is arranged almost along a straight line, a linear relationship is assumed between the quantities entered on the $x$- and $y$-axes.

$$Y = a + bX \quad \text{(IV.1)}$$

The slope $b$ and intercept $a$ are determined if the values of $X$ and $Y$ are given at two suitable points on the straight line. However, a method such as the least squares method is required to obtain more accurate values.

〔**Least squares method**〕 The principle is simple. We want to "determine the straight line that minimizes the sum of the squares of the differences between the measured points and the line that was drawn." For the details, please see the appendix §3 (page 147). Here we only discuss the case where for $x_i$, the measure value of $X$, $y_i$, the measured value of $Y$, has constant error bar ($\equiv \sigma$).

$$a = \frac{1}{\Delta}\left(\sum x_i^2 \cdot \sum y_i - \sum x_i \cdot \sum x_i y_i\right) \tag{IV.2}$$

$$b = \frac{1}{\Delta}\left(n \sum x_i y_i - \sum x_i \cdot \sum y_i\right) \tag{IV.3}$$

where $\Delta$ is given by

$$\Delta = n \sum x_i^2 - \left(\sum x_i\right)^2 \tag{IV.4}$$

and the summation index $i$ is tacitly assumed to run over from 1 to $n$. The errors $\sigma_a, \sigma_b$ of $a, b$ are given by

$$\sigma_a = \sqrt{\frac{1}{\Delta}\sum x_i^2} \cdot \sigma \qquad \sigma_b = \sqrt{\frac{n}{\Delta}} \cdot \sigma \tag{IV.5}$$

$$\sigma \sim \sqrt{\frac{1}{n-2}\sum (y_i - a - bx_i)^2} \tag{IV.6}$$

Since personal computers or electronic calculators possess the function of the above calculation, we can easily determine the coefficients $a, b$.

**b) Semi-log graph** A graph with a normal scale (linear scale) on the horizontal axis and a logarithmic scale on the vertical axis is called a semi-log graph.

The logarithmic scale is obtained if we put the scale mark $y$ at the point with value $\log_{10} y$ on the vertical axis. Note that we never have $y = 0$ on the semi-log graph.

When the following relationship holds between two quantities ($X$, $Y$):

$$Y = a\mathrm{e}^{bX} \tag{IV.7}$$

and if we take the common logarithm ($\log \equiv \log_{10}$) of both sides, we obtain the following relation

$$\log Y = \log a + bX \log \mathrm{e} \tag{IV.8}$$

If we plot $X$ on the horizontal axis and $\log Y$ on the vertical axis, we get a straight line with an intercept of $\log a$ and a slope of $b \log e$. Therefore, a semi-log graph is convenient for graphing and analyzing phenomena having a functional relationship indicated by $Y = ae^{bX}$. Since a scale for $\log Y$ has been assigned on the vertical axis in a semi-log graph, you can directly plot the values of $Y$ rather than calculating the values of $\log Y$ (see Figure IV.2).

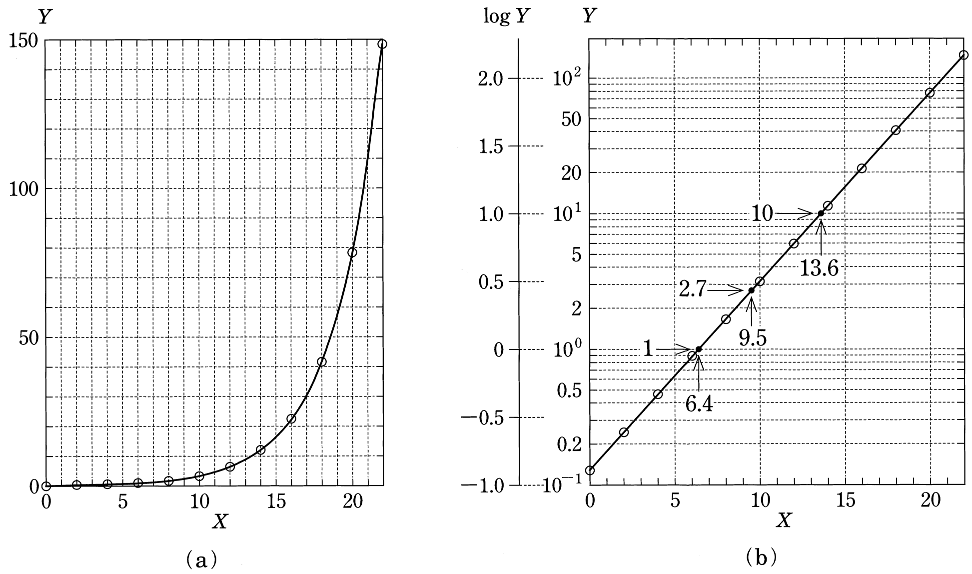



**Fig IV.2** (a) Linear Graph and (b) Semi-log Graph of $Y = 0.13\exp(0.32X)$

On the other hand, when a group of data points are arranged along a straight line on a semi-log graph, you can obtain the empirical formula from the intercept and slope of this graph. That is, obtain a from the value of $Y$ at $X = 0$ or, in other words, from the intercept of the graph. You can also obtain $a$ by substituting any point $(X_1, Y_1)$ on the experimental line into equation (IV.7) after determining $b$ by one of the methods described below. There are several methods of obtaining $b$. Three of these methods are described below.

If we take any two points $(X_1, Y_1)$ and $(X_2, Y_2)$ on the experimental line of the graph, the following relationships will hold.

$$\log Y_1 = \log a + bX_1 \log \mathrm{e}$$

$$\log Y_2 = \log a + bX_2 \log \mathrm{e}$$

Therefore, if we take the difference between these two equations, we obtain the following.

$$b = \frac{1}{\log \mathrm{e}} \frac{\log Y_2 - \log Y_1}{X_2 - X_1} \tag{IV.9}$$

If we substitute the values of $(X_1, Y_1)$ and $(X_2, Y_2)$ into this equation, we can obtain $b$.

2) If we take $(X_2, Y_2)$ so that $Y_2$ is $10Y_1$ in the method described above in a), then equation (IV.10) will be as follows.

$$b = \frac{1}{\log \mathrm{e}} \frac{\log 10Y_1 - \log Y_1}{X_2 - X_1} = \frac{1}{0.4343} \frac{1}{X_2 - X_1} \tag{IV.10}$$

If we read the $X_1$ and $X_2$ corresponding to $Y_1$ and $10Y_1$ from the graph, then we can obtain $b$ from equation (IV.10). In the example shown in Figure IV.2, $b$ is obtained as follows.

$$b = \frac{1}{0.4343}\frac{1}{13.6 - 6.4} = 0.32$$

である.

3) If we take the natural logarithm ($\ln \equiv \log_e$) of both sides of $Y = ae^{bX}$, we obtain the following equation.

$$\ln Y = \ln a + bX \tag{IV.11}$$

Select two points on the experimental line of the graph so that $Y_1 = 10^n$ and $Y_2 = \mathrm{e} \times 10^n = 2.718 \times 10^n$ (where $n$ is any integer) and let $X_1$ and $X_2$ denote the $X$ coordinates corresponding to them. In other words, the two points are $(X_1, 10^n)$ and $(X_2, 2.718 \times 10^n)$. Since these two points each satisfy equation (IV.11), the following two equations hold.

$$\ln 10^n = \ln a + bX_1$$

$$\ln \mathrm{e}10^n = \ln a + bX_2$$

Therefore, if we take the difference between these two equations, we obtain the following.

$$b = \frac{\ln \mathrm{e}10^n - \ln 10^n}{X_2 - X_1} = \frac{1}{X_2 - X_1} \tag{IV.12}$$

If we read the $X_1$ and $X_2$ from the graph, then we can obtain $b$ from equation (IV.12). In the example shown in Figure IV.2, $b$ is obtained as follows.

$$b = \frac{1}{9.5 - 6.4} = 0.32$$

**c) Log-log graph** A graph with a logarithmic scale both on the horizontal axis and on the verticalaxis is called a log-log graph.

When the following relationship holds between two quantities $(X, Y)$:

$$Y = aX^b$$

if we take the common logarithm ($\log \equiv \log_{10}$) of both sides, we obtain the following.

$$\log_{10} Y = \log_{10} a + b \log_{10} X$$

Therefore, the points $(\log_{10} X, \log_{10} Y)$ will fall on a straight line on a log-log graph. Since the value of the power $b$ often has a very important meaning, this is a useful data reorganization method. To obtain an empirical formula from a straight line graph, we should calculate the slope $b$ and intercept $a$. If we assume, for example, that the slope $b$ rises $q$ (cm) when it moves $p$ (cm) sideways, then $b = q/p$, and the intercept $a$ is given by the value of $Y$ at the position where $\log_{10} X = 0$ (which is where $X = 1$). This calculation can be

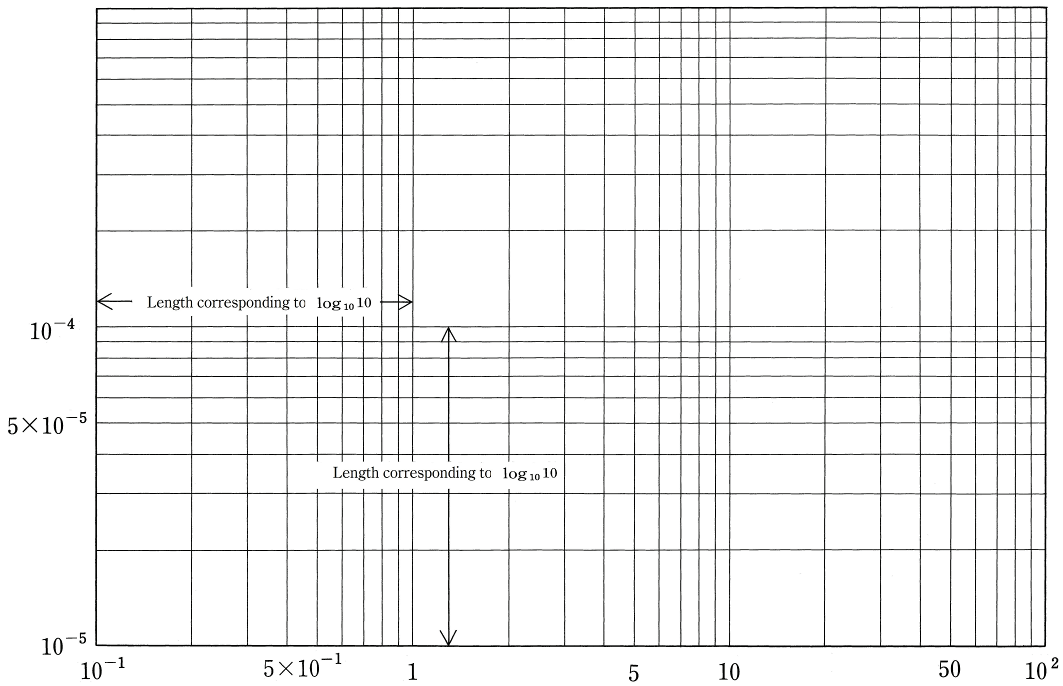



**Fig IV.3** log-log graph

done by a taking a measurement using a regular ruler. If the point where $\log_{10} X = 0$ is not on the graph, it can be obtained by using two data points. Be careful not to confuse the values of $\log_{10} a$ and $a$ when using this procedure. The least squares method described above is certainly useful for determining the coefficients $a$ and $b$ needed to determine the straight line.

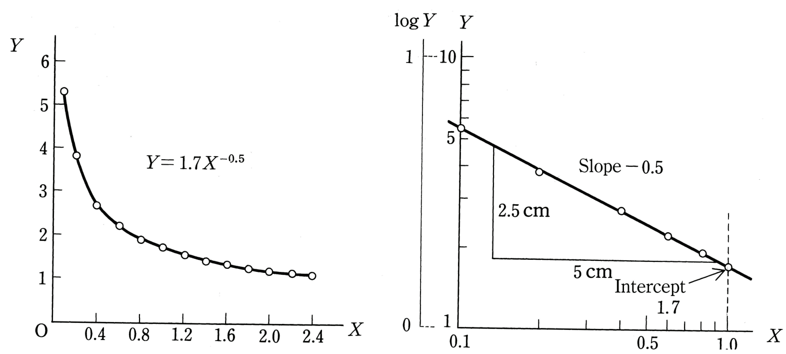



**Fig IV.4** How to Obtain the Power by Using a Log-log Graph

# V. Laboratory Notebook and Experiment Reports

Finishing up the planned measurement does not mean the end of the experiment. Unless you write up the report or paper, the experiment is not over. In order to write the report, we need the laboratory notebook (lab notebook, for short). In the following we explain how to write the lab notebook as well as experimental report.

## §1 Laboratory Notebook

In order to summarize the results of the research in the paper or in the report, it is essential and indispensable to record anything what you have encountered in the **laboratory notebook** and insure the reproducibility. If you have obtained a certain result, you have to repeat the measurement and reproduce the same result so that you confirm the condition of the experiment. In some case, however, you may get an unexpected result. In such a case, you have to examine what are the important conditions to reproduce the same result or what are the different conditions which leads to the different results based on the laboratory notebook. Namely the recording in the laboratory notebook is extremely important to perform the experiment and it is not too much to say that it determines success or failure of the experiment. Depending on the research field, but sometimes the description of the laboratory notebook affects whether the patent can be acquired or not. In some case, in order for the research result not to be brought outside withoutpermission, notebooks are strictly put under control.

An example of laboratory notebook is shown in Fig.V.1. Notebooks sewn by thread without fear of break up are strongly recommended. If the grids of the size about 5mm are printed, it is convenient to draw a simple figure and a graph. Instead of an ordinary pencil, you should use a ballpoint pen which is not easy to erase. A written mistake can be deleted by a double line. Note that it is sometimes important to keep mistakes on record. It may happen that what was thought to be a mistake turns out to be correct afterwards. So the year/month/date of the experiment and the middle stage of the calculation should be also recorded. In doing so we can specify at what stage we made a mistake and we can share the information among the collaborators.

2016 Mo. Day.

13:00

Experiment of Coupled Oscillation

Humidity: % Temperature： C Weather： Atmospheric Pressure： h Pa

Lab partner： Table No.

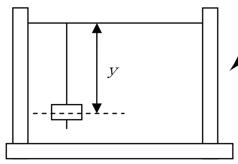
A. Characteristic of Simple Pendulum

13:15

Objective: Measuring the variation of frequency with respect to arm length and compare with frequency of the ideal pendulum

Keep a record of Date and Time

Draw a figure for later use




Experimental Apparatus

Arm length is measured in the center of the weight

Write precaution or what you have noticed
（Keep a space for that purpose）

2. Method of experiment A

Choose a weight of 0.35 kg

Varying the distance between the center of the weight and the rotation axis, we measure the frequency of the pendulum (using the scale line of the SUS pipe). Fixing the arm of the pendulum to the rotation axis, we keep free both end of the axis on the bearing. Taking the initial amplitude to x=5 cm and measuring the time from the resting state to the 30th period, we obtain the frequency from the equation f=1/T, where T is the period.

Delete mistakes by double line

Result for various arm-lenght with fixed weight 0.35kg

| mass $m$[kg] | length $y$[m] | oscillation $N$ | time [s] | period $T$[s] | frequency $f$[Hz] | $f_{model}$[Hz] |
|---|---|---|---|---|---|---|
| 0.35 | 0.3 | ~~30~~ 32 | … | … | … | … |
| 0.35 | 0.35 | 30 | … | … | … | … |
| … | | | | | | |
| … | | | | | | |

Changing weight from 0.35 kg to 0.175 kg

13:40

Result for various arm-lenght with fixed weight 0.175kg

| mass $m$[kg] | length $y$[m] | oscillation $N$ | time [s] | period $T$[s] | frequency $f$[Hz] | $f_{model}$[Hz] |
|---|---|---|---|---|---|---|
| ~~0.35~~ 0.175 | 0.3 | 30 | … | … | … | … |
| … | | | | | | |

**Fig V.1** An example of laboratory notebook

## §2 Experiment Reports

You create an experiment report to communicate to other people the legitimacy of the experiment you performed and the importance of the conclusions you obtained. Therefore, it is important for you to clearly and concisely represent to other people what you have done with attention to the main points and to try to write the report so that other people will be able to read it. It is also meaningful for your actions to appeal to other people. A report should not be long.

The work of creating a report does not start after the experiment is finished. Throughout the entire process of performing the experiment, it is important for you to verify whether the experiment is proceeding without mistake and whether or not you are measuring correctly. A reliable and ultimately efficient way of accomplishing this is to draw a rough graph of the measured values while the experiment is proceeding. If the trend represented by the graph diverges from the estimation, you may have made a new discovery, but it often indicates that you may have made a mistake in an operation or that a device may have malfunctioned. If you perform the experiment while estimating the next data, you can form your own image concerning the subject. That will be the starting point for your discussion and considerations and may help in obtaining a point of view when writing your report.

A report is written divided broadly into the following items.

1. Objectives of the Experiment
2. Principles of the Experiment
3. Experimental Devices
4. Experimental Method
5. Measured Data
6. Analysis and Results
7. Discussion and Considerations
8. Conclusion

**1) Objective of the Experiment** Specifically describe in several lines of text what you are seeking in this experiment and how you intend to find it.

**2) Principles of the Experiment** Describe why the objective described above will be reached by this experiment as well as the physical contents of the experiment. Present a theoretical explanation and describe required relational expressions. At this time, be sure not to copy verbatim from your textbook. Instead, understand what it says and briefly describe it in your own words. A textbook is for teaching someone who does not know something, while a report is for communicating to another person the things you have done. Think about these differences. For example, you need not include explanations of physics terms in a report.

**3) Experimental devices** Describe all experimental devices you used and be sure to number those devices. Also, describe how those devices were organized, draw wiring diagrams, etc.

**4) Experimental method** Briefly describe what method you used to perform the experiment with a focus on the setup conditions of the devices. Also consider the differences between a textbook and a report here. You need not copy verbatim from the textbook.

**5) Measured data** First, organize and indicate the raw data that you obtained from measurements in a form that is easy to read.

There are several points you must keep in mind when dealing with measured values. First, do not forget units. Physical quantities always must have units. For example, if a length is expressed as just 2.5, you do not know whether it means 2.5 m or 2.5 cm. The next problem is the precision of measured values and significant digits. For example, 1 m is incorrect when measurements were taken in terms of mm. You must write 1.000 m. Also, when the same measurement is repeatedly made, the results should not appear to have varying measurement precisions such as 1.21, 1.2, and 1.225.

Later, create the tables and figures (graphs) required for analysis. Be sure to assign sequence numbers to the tables and figures (graphs) in the report as Table 1, Table 2, ···, Figure 1, Figure 2, ··· and assign titles. Clearly describe the correspondence with a table or graph in the main text as well as the point in the main text at which the reader should look at the table or graph. Provide appropriate guidance to each table or figure in an easy-to-read form when the main text is read sequentially from beginning to end. A table or graph should not appear alone without any explanation. You should explain what you measured to obtain the measured values and the conditions under which you obtained them. Provide clear explanations in the text of what is being graphed in each numbered figure, what table data was used to graph it, how you drew the curve, and so on.

See I-§3 for information about drawing figures. It is important for a figure to be easy to view. Unlike in tables or the main text, numbers accompanying the scales on the vertical and horizontal axes of a figure should be as brief as possible (for example, $1.00 \to 1$, $0.100 \to 0.1$).

Do not cut the graph paper into small pieces and glue them to the report paper.

**6) Analysis and results** The first point you should carefully consider when using measured data to obtain results according to calculations is the number of significant digits. If measured data having only three digits is used for calculations on a calculator and you directly write the full number of digits displayed on the calculator in your report, you will fail as an experimental researcher. Refer to the bottom of page 1 of this textbook and be careful concerning significant digits.

To indicate the mean value of measured data, refer to I-§2 to obtain the standard deviation and express measured results as $\overline{X} \pm \sigma_{\text{exp}}$. In addition, enter error bars in graphs if possible.

**7) Discussion and Considerations** After viewing the tables and figures as a whole, add your considerations. The contents will vary according to the topic. It is traditional to objectively and scientifically describe what you learned from the experimental results and what concepts occur to you based on them.

If the topic is to determine a certain physical quantity, represent the results that you obtained from the experiment you performed, for example, as (mean value) ± (standard deviation). Allowing some latitude is extremely important in an experiment.

In addition, investigate whether or not the reference values or theoretical values that appear in the textbook fall within that range. If there is a large difference, consider reasons why that happened. You should also refer to equation ① in I-§1 to quantitatively check which element has the largest error in the experiment. You should also write that you studied about the description in the textbook. Describe the investigation results in your own way, especially when writing about the topic of investigation.

**8) Conclusion**

**[Precautions]**

① Except when specifically indicated otherwise, you should create one report for each experiment and submit it at the prescribed location. It should be delivered by 1:00 pm on the day of the following week's experiment. Any report that is submitted late will receive demerits.

② For report cover sheets, use the pages that are included in the appendix of this textbook. Copies will not be accepted.

③ For report pages and graph paper, use paper that is the same B5 size as the cover sheets. Do not use loose leaf paper.

④ Each person must provide their own millimeter graph paper. You must always bring it when performing experiments and plot the data on site. If you realize your mistake when organizing data at home, it will be too late.

⑤ For the day-of-the-week field of the cover sheet, enter the day of the week that was assigned in class. For the group number, enter the number that was assigned when the class was divided into groups. Do not forget to enter the name of your lab partner.

⑥ Staple together the cover sheet, graphs, etc. included in the report. Do not use a paper clip since there is a risk that pages may be lost.

⑦ If you are absent on the specified day for an experiment, you must talk to the teacher in charge. If you have not attended the stipulated number of experiments, you may be unable to receive credit for the course.

⑧ When your report is returned, if you are instructed to resubmit the report, redo the experiment, or meet for a required interview, follow that instruction quickly.

# Experiment 1. Coupled Oscillations

## §1 Experimental Objective

Coupled oscillators interact with each other through the torsion of a metal rod. In this chapter, we cover three subjects that verify the model of coupled oscillations.

**A. Characteristics of a Simple Pendulum**

Measure the dependency of frequency with length of the pendulum, and compare experimental data with that of an ideal pendulum.

**B. Hooke' Law for Torsion**

Determine the relation between torsion angle and torque of the metal shaft, to estimate the interaction strength between the two pendulums.

**C. Characteristics of Coupled Oscillations**

Measure the normal mode frequencies of the coupled oscillators system, and confirm that coupled oscillations can be expressed by superposition of these normal modes.

## §2 Experimental Apparatus

Figure 1.1 shows the configuration of the experimental apparatus used in this topic. The pendulum consists of a shaft made from a light stainless steel (SUS) pipe (diameter: 7 mm; thickness: 0.5 mm; length: 632 mm) and a 0.35 kg iron weight (diameter: 60 mm; thickness: 16 mm). The weight can be interchanged with 0.70 kg or 0.175 kg for comparison. The effective length of the pendulum arm can be changed by moving the position of the weight along the stainless steel pipe and affix it with a screw. To help with setting the arm length, lines are engraved at 25 mm intervals along the stainless steel pipe.

A round stainless steel rod with a diameter of 2 mm is used as the axis of rotation of the pendulum. To enable free rotation of the rod without twisting and with little friction, knife-edge bearings are affixed to the beam with a spacing of 50 mm. The round rod is placed on the knife-edge bearings and a metal fitting at the top of the pendulum arm is affixed to the rod with a screw at an intermediate point of the knife-edge. Either one or two pendulums can be mounted on the axis of rotation. The spacing $\Delta z$ between the two bearings (pendulum positions) is 0.5 m, as shown in Figure 1.1.

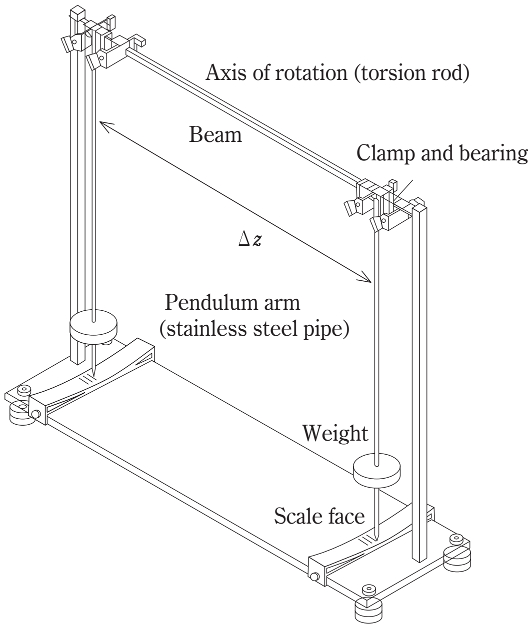



**Fig 1.1** Experimental Apparatus of Coupled Oscillations

To measure the pendulum angle $\theta$, first measure the length $x = s - s(0)$ obtained by subtracting the scale graduation $s(0)$ indicating where the pendulum is at rest from the scale graduation s indicating where the pointer attached to the tip of the arm points on the curved surface ruler. Since this scale is demarcated in cm units, the angle $\theta = x/R$ can be determined if this length is divided by the distance $R = 63.5$ cm from the axis to the ruler. A stopwatch is used to measure time.

## §3 Experiment A: Simple Oscillation Characteristics

### 1. Experiment A: Concepts

If $h$ [m] denotes the distance from the axis of rotation to the center of gravity of the pendulum, $m$ [kg] denotes the mass of the pendulum, and $I$ [kg·m$^2$] denotes the moment of inertia of the pendulum with respect to the axis of rotation, then the deflection angle $\theta$ of the pendulum is represented as follows:

$$I\frac{d^2\theta}{dt^2} = -\,mgh\sin\theta$$

The left side represents the magnitude of the inertia of the rotational motion, and the right side represents the restoration torque due to gravity. If the amplitude is small, then $\sin\theta$ can be well approximated by $\theta$ and this equation of motion can be written as follows:

$$I\frac{d^2\theta}{dt^2}=-mgh\theta \tag{1.1}$$

Since this represents that a particle of mass $I$ attached to a spring with spring constant $mgh$ receives a force of $-mgh\theta$ and oscillates with acceleration $\frac{d^2\theta}{dt^2}$, its angular frequency $\omega$ [rad/s] is given as follows:

$$\omega=\sqrt{\frac{mgh}{I}} \tag{1.2}$$

If $\theta_0$ denotes the initial value of the deflection angle and the oscillation begins from a state of rest, then the deflection angle $\theta$ at time $t$ can be represented as follows:

$$\theta=\theta_0\cos(\sqrt{\frac{mgh}{I}}t) \tag{1.3}$$

The oscillation frequency $f$ [Hz] is given as follows:

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{mgh}{I}} \tag{1.4}$$

## 2. Method for Experiment A

If we neglect all masses other than that of the weight, the observed frequency should match that of a simple oscillator. Select the 0.35 kg (16 mm thickness) weight. You will measure the frequency $f$ of the pendulum while using the engraved lines on the stainless steel pipe to change the distance y between the center of the weight and rotation axis. Affix the pendulum arm to the rotation axis and let it hang freely by just placing both ends of that axis on the bearings. Set the initial amplitude to $x = 5$ cm and measure the time required for 30 cycles (or more) from a state of rest to obtain both the time $T$ required for 1 round trip and the frequency $f = \omega/2\pi = 1/T$ from its reciprocal. Record the data you obtain in the format shown in Table 1.1. Change $y$ from 0.3 m to 0.6 m in steps of approximately 5 cm. Assume that the entire mass is concentrated in the center of the weight and use the gravitational acceleration for Kyoto $g = 9.797$ m/s$^2$ and the arm length $y$ to add the frequency of an ideal pendulum $f_{\mathrm{model}}$ to the table, which is given as follows:

$$f_{\mathrm{model}}=\frac{1}{2\pi}\sqrt{\frac{g}{y}} \tag{1.5}$$

We then plot $f$ and $f_{\mathrm{model}}$ as Figure A, an example of which is shown in Figure 1.2.

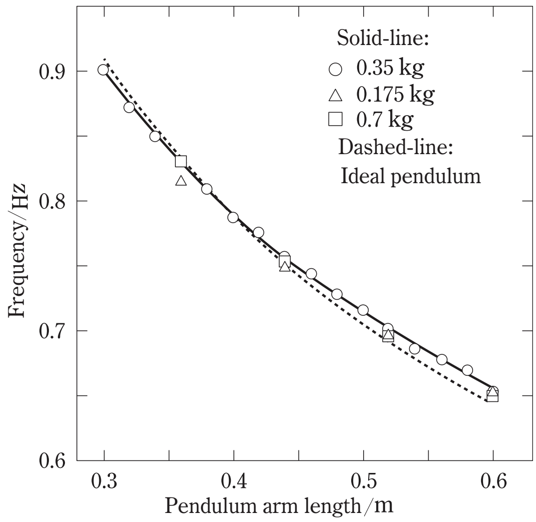



**Fig 1.2** Dependence of Frequency on Pendulum Arm Length. Corresponds to Figure A.

### 3. Analysis Tasks for Report

1. Plot the deviation $\Delta f$ between the observed frequency and that of a simple oscillator, as a function of the arm length $y$.
2. Discuss qualitatively reasons for the observed deviation.

**Table 1.1** Frequency of a Single Physical Pendulum

| Mass<br>$m$ [kg] | Arm length<br>$y$ [m] | No. of Oscillation<br>$N$ | Observation time<br>[s] | Period<br>$T$ [s] | Frequency<br>$f$ [Hz] | Model Frequency<br>$f_{\mathrm{model}}$ [Hz] |
|---|---|---|---|---|---|---|
| 0.35 | 0.3 | ⋯ | ⋯ | ⋯ | ⋯ | ⋯ |
| ⋯ | ⋯ | ⋯ | ⋯ | ⋯ | ⋯ | ⋯ |

## §4 Experiment B: Hooke's Law and Coupling Coefficient Measurement

### 1. Experiment B Concepts

For a simple pendulum, no torsion occurs since the axis rotates freely together with the pendulum. However, if an external force is applied to intentionally rotate the axis, the pendulum that is affixed to the axis will slant at an angle of $\theta$ from the vertical position. As a result, the moment of force (torque) due to gravity is added to the rotation axis through the pendulum arm. The deflection angle is determined when the torque of the external force $N$ and the gravitational torque $mgh \sin\theta$ are in equilibrium. In other words, it is determined when the following equation holds:

$$N = mgh \sin\theta \tag{1.6}$$

The $h$ that is introduced here is the effective length of the pendulum including the contribution of the mass distribution of the arm. If the mass of the arm can be ignored, this is equal to the actual length $y$ from the axis to the center of the weight. We will ad a second pendulum here to a common axis to create a linked coupled pendulum system in which two pendula (let them be A and B) are separated by a distance of $\Delta z$. When the two pendula are left alone to hang naturally, they face in the vertical direction and there is no torsion between them. If you grasp pendulum A with your hand and slowly rotate it to bring it to the position with angle $\theta_A$, pendulum B will be drawn towards it and will slant at angle $\theta_B$. Torque $N$, which causes pendulum B to slant, arises from the difference in angles $\Delta\theta = \theta_A - \theta_B$ that occurs between the two pendula. If the torsion angle $\Delta\theta$ is sufficiently small compared to $\pi$, then a proportional relationship can be considered to hold between $N$ and $\Delta\theta$. Let's introduce the coupling coefficient $c$ and represent this relationship as follows:

$$N = c\,(\theta_A - \theta_B) \tag{1.7}$$

If we apply equation (1.6) for torque equilibrium to pendulum B and assume the relationship in equation (1.7), then we can expect the following relationship to hold:

$$\theta_A - \theta_B = \frac{m_B g h_B}{c} \sin\theta_B \tag{1.8}$$

This is Hooke's law for torsion. Figure 1.4 shows an example of experimental result for an arm length $y_B = 0.45$ m and $\Delta z = 0.5$ m. The topic of this experiment is to quantitatively assess this relationship.

## 2. Method for Experiment B

Affix pendula A and B, which both have a 0.35 kg weight and an arm length of $y = 0.45$ m, to the rotation axis with a spacing between them of $\Delta z = 0.5$ m. When affixing them to the rotation axis, make sure there is no torsion. This is important for increasing the precision of the experiment. First, use a screwdriver to tightly affix only one pendulum to

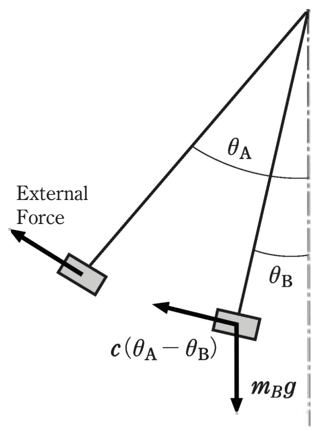



**Fig 1.3** Experiment Method of Hooke's Law for Torsion

the rotation axis. Place the other pendulum on the axis so that it is hand-tightened and can rotate without making contact. Leave the pendula so they are hanging down parallel to each other and then tighten the hand-tightened pendulum with the screwdriver. Record the scale graduations $s_A(0)$ and $s_B(0)$ indicated by the pointer in this state where there is no external force. Accurately measure $\Delta z$ and arm length $y_B$ of pendulum B and record them. Grab the weight of pendulum A with your hand and slowly increase the scale graduation $s_A$ indicated by the pointer a little at a time to change the inclination angle $\theta_A = (s_A - s_A(0))/R$ as shown in Figure 1.3. For each of those $s_A$ values, read the scale graduation $s_B$ indicated by the pointer of pendulum B to measure inclination angle $\theta_B = (s_B - s_B(0))/R$. Record the data in the format shown in Table 1.2. For each measurement, enter the data both in the table and a graph, as this is effective for increasing the efficiency of the experiment and eliminating errors.

### 3. Analysis Tasks for Report

1. Take $\theta_B \sim \sin\theta_B$ for the horizontal axis and $\theta_A - \theta_B$ for the vertical axis and draw a graph (label it Figure B) from the results in Table 1.2. Draw a straight line (called the regression line) that best collects together the data points overall on this graph and determine its slope $q$ (see subsection 3 of "Graphs and Data Analysis" on page 24).
2. In addition, obtain the upper and lower bounds of the slope and plot them as shown in Figure 1.4. Provide a numerical analysis of the uncertainty.

$$q = \frac{m_B g h_B}{c} = \text{Best value of slope} \pm \text{slope margin} \tag{1.9}$$

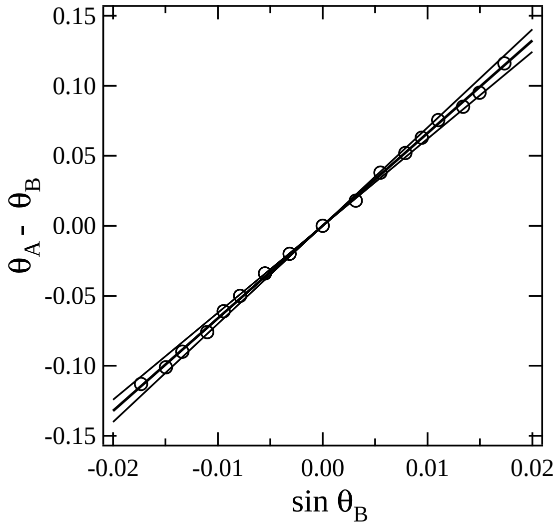



**Fig 1.4** Example of Experimental Results of Hooke's Law for Torsion. Corresponds to Figure B.

**Table 1.2** Relationship of the Difference in Angles $\theta_A$ - $\theta_B$ of the Two Pendula and Inclination Angle $\theta_B$

| $\Delta z$/cm = | $y_B$/cm = | $m_B$/kg = | $s_A(0)$/cm = | $s_B(0)$/cm = | |
|---|---|---|---|---|---|
| $s_A$/cm | $\theta_A$ | $s_B$/cm | $\theta_B$ | $\sin\theta_B$ | $\theta_A$ - $\theta_B$ |
| -8 | -0.126 | ··· | ··· | ··· | ··· |
| -7 | -0.110 | ··· | ··· | ··· | ··· |
| ··· | ··· | ··· | ··· | ··· | ··· |

## §5 Experiment C: Coupled Oscillation

### 1. Experiment C Concepts

#### Motion of Coupled Oscillators

Let us consider, as in the experiment B, the motion of two pendula with the same mass and the same arm-length, and mutually linked by an axis. A restoring force moment due to torsion that arises from the moment of gravity and difference in deflection angles acts on each pendula A and B. If we denote the mass of each pendulum by $m$, the moments of inertia by $I$, and the effective arm length by $h$, then the two pendula are represented by the following equations of motion.

$$I\frac{d^2\theta_\mathrm{A}}{dt^2} = -\,mgh\sin\theta_\mathrm{A} + c\,(\theta_\mathrm{B} - \theta_\mathrm{A})$$

$$I\frac{d^2\theta_\mathrm{B}}{dt^2} = -\,mgh\sin\theta_\mathrm{B} + c\,(\theta_\mathrm{B} - \theta_\mathrm{A}) \tag{1.10}$$

Assume the deflection angles are small and use the approximations, $\sin\theta_\mathrm{A} = \theta_\mathrm{A}, \sin\theta_\mathrm{B} = \theta_\mathrm{B}$. Also, to simplify the notation, let $\omega_0 = \sqrt{mgh/I}, \gamma = \sqrt{c/I}$, so that these equations of motion can be rewritten as follows:

$$\frac{d^2\theta_\mathrm{A}}{dt^2} = -\,(\omega_0^2 + \gamma^2)\theta_\mathrm{A} + \gamma^2\theta_\mathrm{B} \tag{1.11}$$

$$\frac{d^2\theta_\mathrm{B}}{dt^2} = -\,(\omega_0^2 + \gamma^2)\theta_\mathrm{B} + \gamma^2\theta_\mathrm{A} \tag{1.12}$$

The mutual positional coordinates in the equations of motion when two pendula are linked are complicated and cannot be solved in this form. By the change of the variables, (Eq.(1.11)+Eq.(1.12))/2 and (Eq.(1.11)-Eq.(1.12))/2 given as

$$\theta_\mathrm{G} = \frac{\theta_\mathrm{A} + \theta_\mathrm{B}}{2}, \qquad \theta_\mathrm{R} = \frac{\theta_\mathrm{A} - \theta_\mathrm{B}}{2} \tag{1.13}$$

the above coupled equations are reduced to the following equations of motion for the two independent simple oscillations,

$$\frac{d^2\theta_\mathrm{G}}{dt^2} = -\,\omega_0^2\theta_\mathrm{G}, \qquad \frac{d^2\theta_\mathrm{R}}{dt^2} = -\,(\omega_0^2 + 2\gamma^2)\theta_\mathrm{R} \tag{1.14}$$

Note that $\theta_\mathrm{G}$ means the center of mass coordinate for $\theta_\mathrm{A}$ and $\theta_\mathrm{B}$, while $\theta_\mathrm{R}$ stands for a half of the relative coordinates of them. If we write

$$\omega_1 = \omega_0, \qquad \omega_2 = \sqrt{\omega_0^2 + 2\gamma^2} \tag{1.15}$$

then we obtain $\theta_\mathrm{G}, \theta_\mathrm{R}$ as the general solutions of simple oscillation given by

$$\theta_\mathrm{G} = A\cos(\omega_1 t + \phi_1), \qquad \theta_\mathrm{R} = B\cos(\omega_2 t + \phi_2) \tag{1.16}$$

where $A, B, \phi_1, \phi_2$ are integration constants. We call these two simple oscillations as **normal modes**. From the change of variables (1.13), we can recover $\theta_\mathrm{A}, \theta_\mathrm{B}$ as the general solutions

$$\theta_\mathrm{A} = A\cos(\omega_1 t + \phi_1) + B\cos(\omega_2 t + \phi_2)$$

$$\theta_\mathrm{B} = A\cos(\omega_1 t + \phi_1) - B\cos(\omega_2 t + \phi_2) \tag{1.17}$$

Thus we find that the motions of the two coupled pendula can be expressed as the **superposition of normal modes**.

**Normal mode and beat**

Let us consider the three intial conditions.

1. At $t = 0$ the deflection angles are $\theta_A = \theta_B = \theta_0$, and both pendula are at rest. More explicitly we have

$$\theta_A = \theta_0, \ \ \theta_B = \theta_0, \ \ \frac{d\theta_A}{dt} = 0, \ \ \frac{d\theta_B}{dt} = 0$$

Thus we find the integration constants are determined as $A = \theta_0, B = 0, \phi_1 = \phi_2 = 0$, and we get

$$\theta_A = \theta_0 \cos(\omega_1 t), \qquad \theta_B = \theta_0 \cos(\omega_1 t) \tag{1.18}$$

Hence we note that the above initial conditions lead to the **normal mode of the angular frequency** $\omega_1$. In this normal mode, the amplitudes of $\theta_A, \theta_B$ are the same (or they are in the same phase).

2. At $t = 0$ the deflection angles are $\theta_A = \theta_0, \theta_B = -\theta_0$, and both pendula are at rest. In this case the integration constants are determined as $A = 0, B = \theta_0, \phi_1 = \phi_2 = 0$. Thus we have

$$\theta_A = \theta_0 \cos(\omega_2 t), \qquad \theta_B = -\theta_0 \cos(\omega_2 t) \tag{1.19}$$

Hence we note that the above initial conditions lead to the **normal mode of the angular frequency** $\omega_2$. In this normal mode, the amplitudes of $\theta_A, \theta_B$ are the same in their magnitude but with opposite sign. (or they are in the opposite phase).

3. At $t = 0$ the deflection angles are $\theta_A = \theta_0, \theta_B = 0$, and both pendula are at rest. In this case the integration constants are determined as $A = B = \theta_0/2, \phi_1 = \phi_2 = 0$. Thus we have

$$\begin{aligned} \theta_A &= \frac{\theta_0}{2}\{\cos(\omega_1 t) + \cos(\omega_2 t)\}, \\ \theta_B &= \frac{\theta_0}{2}\{\cos(\omega_1 t) - \cos(\omega_2 t)\} \end{aligned} \tag{1.20}$$

Rewriting the solutions by the formula for the trigonometric functions we get

$$\begin{aligned} \theta_A &= \theta_0 \cos(\frac{\omega_1 + \omega_2}{2} t) \cos(\frac{\omega_2 - \omega_1}{2} t), \\ \theta_B &= \theta_0 \sin(\frac{\omega_1 + \omega_2}{2} t) \sin(\frac{\omega_2 - \omega_1}{2} t) \end{aligned} \tag{1.21}$$

Thus the amplitudes are varying with the time (Amplitude Oscillation). The period of the amplitude oscillation can be expressed as

$$T_{AB} = \frac{2\pi}{\omega_2 - \omega_1} \tag{1.22}$$

When $\omega_2$ is close to $\omega_1$, this period becomes long, and the oscillations for the two pendula are found to show **beat**.

## 2. Method for Experiment C

### Experimental Conditions

We fix the weights of the two pendula to the same mass $m = 0.35$ kg and set the spacing between them $\Delta z$, as in experiment B, to $\Delta z = 0.5$ m. The arm lengths of both pendula are taken to the same value, $y = 0.45$ m. The amplitude of the oscillators are measured by the scales $s_\mathrm{A}$ and $s_\mathrm{B}$. We study the motion of the pendula for the following three initial conditions.

(a) Initial condition giving the normal mode 1 (angular frequency $\omega_1$)
At $t = 0$, we place the pointers at the positions, $s_\mathrm{A}$=5 cm and $s_\mathrm{B} = 5$ cm, and gently remove your hand from the state of rest.

(b) Initial condition giving the normal mode 2 (angular frequency $\omega_2$)
At $t = 0$, we place the pointers at the positions, $s_\mathrm{A}$=5 cm and $s_\mathrm{B} = -5$ cm, and gently remove your hand from the state of rest.

(c) Initial condition giving the beat
At $t = 0$, we place the pointers at the positions, $s_\mathrm{A}$=5 cm and $s_\mathrm{B} = 0$ cm, and gently remove your hand from the state of rest.

**Experiment · Measurements**

We measure the eigen frequency for the initial conditions (a), (b), as well as the frequency of amplitude oscillation for the initial condition (c).

(1) For the initial condition (a), the pendula A, B oscillate in the same phase. Determine the frequency $f_1 = \omega_1/(2\pi)$, by measuring the time necessary for the 30 periods, as in the Experiment A. For the initial condition (b), the pendula A, B oscillate in the opposite phase. In the similar manner, determine the frequency $f_2 = \omega_2/(2\pi)$. The data should be recorded in the form of Table 1.3.

(2) For the initial condition (c), we measure the period of the amplitude oscillation. The pendulum B will come to rest periodically. Repeatedly bring it to a state of rest several times and determine the period of the amplitude oscillation $T_\mathrm{AB}$ from the iteration count and required times. The data should be recorded in the form of Table 1.3.

**Points of caution**

Be careful not to move the pendula toward the direction of the axis while removing your hand from the state of rest. It is extremely important to start the oscillation of both pendula simultaneously. When you carry out the experiment in a pair, you should practice enough together with your partner. Pulling the pendula by grabing with your fingers causes unwanted oscillation. You can avoid this kind of trouble, by pushing two weights with a smooth and thick plate from one side at the position of rest and then remove it at once.

**Table 1.3** Normal mode and amplitude oscillation of two coupled oscillators

| $\Delta z$/cm = | $s_A(0)$/cm | $s_B(0)$/cm | # of Osc. $N$ | Obs. time /s | $T_{1,2}, T_{AB}$/s | $f_{1,2}, f_{AB}$/Hz |
|---|---|---|---|---|---|---|
| Ini. Cond. 1 | 5.0 | 5.0 | | | | |
| Ini. Cond. 2 | 5.0 | -5.0 | | ··· | ··· | |
| Ini. Cond. 3 | 5.0 | 0.0 | | ··· | ··· | |

### 3. Analysis Tasks for Report

1. Explain qualitatively the reason why the frequencies of normal modes $f_1, f_2$ satisfy $f_1 < f_2$.
2. Show that the relation between $T_{AB}$ and $q$ is given by

$$T_{AB} = \frac{2\pi}{\omega_0} \frac{1}{\sqrt{1+2/q} - 1}$$

   Then estimate $T_{AB}$ using the value $q$ obtained in the experiment B based on the above equation. In the estimation, take into account the error bar of $q$ as given by Eq.(1.9). Moreover, compare this estimated value with that of $T_{AB}$ obtained in the experiment C, and make sure if they are in agreement within the error bar.
3. In the case of the initial condition (c), draw a graph showing the time dependence of the deflection angles of pendula A and B, based on the measured data $f_1, f_2, f_{AB}$.
4. In this experimental subject we studied the oscillations of coupled pendula. Show an example of coupled oscillation for electric circuits or some other mechanical systems, and explain briefly the characteristics of the normal mode of oscillation.

## §6 Further study — General solution of coupled oscillation —

In Experiment C, we have treated the coupled oscillators in which the two pendula have the same mass and the same arm-length. In the following we discuss more general case where the masses and arm-lengths are different. Let the mass of each pendulum $m_A, m_B$, the moments of inertia $I_A, I_B$, and the effective arm lengths $h_A, h_B$, then we can express the coupled equations of motion as follows:

$$I_A \frac{d^2\theta_A}{dt^2} = -\, mgh_A \sin\theta_A + c\,(\theta_B - \theta_A) \tag{1.23}$$

$$I_{\mathrm{B}} \frac{d^2\theta_{\mathrm{B}}}{dt^2} = -\, mgh_{\mathrm{B}} \sin\theta_{\mathrm{B}} - c\,(\theta_{\mathrm{B}} - \theta_{\mathrm{A}}) \tag{1.24}$$

Assuming that the deflection angles are small, we can approximate $\sin\theta_{\mathrm{A}} = \theta_{\mathrm{A}}, \sin\theta_{\mathrm{B}} = \theta_{\mathrm{B}}$. In order to simplify the notation further, we write $\omega_{\mathrm{A}} = \sqrt{mgh_{\mathrm{A}}/I_{\mathrm{A}}}, \omega_{\mathrm{B}} = \sqrt{mgh_{\mathrm{B}}/I_{\mathrm{B}}}, \gamma_{\mathrm{A}} = \sqrt{c/I_{\mathrm{A}}}, \gamma_{\mathrm{B}} = \sqrt{c/I_{\mathrm{B}}}$. Then we can rewrite the equation of motion as follows:

$$\frac{d^2\theta_{\mathrm{A}}}{dt^2} = -\,(\omega_{\mathrm{A}}^2 + \gamma_{\mathrm{A}}^2)\theta_{\mathrm{A}} + \gamma_{\mathrm{A}}^2\theta_{\mathrm{B}} \tag{1.25}$$

$$\frac{d^2\theta_{\mathrm{B}}}{dt^2} = -\,(\omega_{\mathrm{B}}^2 + \gamma_{\mathrm{B}}^2)\theta_{\mathrm{B}} + \gamma_{\mathrm{B}}^2\theta_{\mathrm{A}} \tag{1.26}$$

To solve these coupled differential equations, we assume the solutions of normal modes as

$$\theta_{\mathrm{A}} = A\cos(\omega t + \phi), \qquad \theta_{\mathrm{B}} = B\cos(\omega t + \phi) \tag{1.27}$$

and substitute them into the above equations, we get the following coupled eqauations for $A$,$B$:

$$(\omega^2 - \omega_{\mathrm{A}}^2 - \gamma_{\mathrm{A}}^2)\,A + \gamma_{\mathrm{A}}^2 B = 0 \tag{1.28}$$

$$\gamma_{\mathrm{B}}^2 A + (\omega^2 - \omega_{\mathrm{B}}^2 - \gamma_{\mathrm{B}}^2)\,B = 0 \tag{1.29}$$

In order for the above equations have non-trivial solution except for $A = B = 0$, it is necessary for the 2×2 matrix of coefficients not to have an inverse matrix. In this case, the determinant of the matrix must vanish and we obtain the following equation to determine the angular frequency $\omega$.

$$(\omega^2 - \omega_{\mathrm{A}}^2 - \gamma_{\mathrm{A}}^2)\,(\omega^2 - \omega_{\mathrm{B}}^2 - \gamma_{\mathrm{B}}^2) - \gamma_{\mathrm{A}}^2\gamma_{\mathrm{B}}^2 = 0 \tag{1.30}$$

By solving this equation we get, as the positive solutions, two frequencies. $\omega_1, \omega_2 (\omega_1 < \omega_2)$. Moreover, the following relations are obtained from the coupled equations (1.28),(1.29):

$$\frac{A}{B} = -\frac{\omega^2 - \omega_{\mathrm{B}}^2 - \gamma_{\mathrm{B}}^2}{\gamma_{\mathrm{B}}^2} = -\frac{\gamma_{\mathrm{A}}^2}{\omega^2 - \omega_{\mathrm{A}}^2 - \gamma_{\mathrm{A}}^2} \tag{1.31}$$

By substituting $\omega = \omega_1, \omega_2$ into the above equation, we get the ratio of the amplitudes of normal modes $A/B$. Writing the amplitudes for $\omega_1$ ($\omega_2$) as $A_1, B_1$ ($A_2, B_2$), the general soltutions for $\theta_{\mathrm{A}}, \theta_{\mathrm{B}}$ are given by a superposition of the nomal modes as follows:

$$\begin{aligned}\theta_{\mathrm{A}} &= A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) \\ \theta_{\mathrm{B}} &= B_1\cos(\omega_1 t + \phi_1) + B_2\cos(\omega_2 t + \phi_2)\end{aligned} \tag{1.32}$$

where $\phi_1, \phi_2$ denote the phase differences. Since the ratio of the amplitudes $A/B$ are determined for each normal mode, the unknown constants are $A_1, A_2, \phi_1, \phi_2$. Those four unknown constatnts are determined from the initial conditions (the initial positions and velocities of $\theta_{\mathrm{A}}, \theta_{\mathrm{B}}$).

# Experiment 2. Measurement of Electrical Resistance

## §1 Experimental Objective

We measure the electrical resistance of a metal, semiconductor, and superconductor while lowering the temperature with ice or liquid nitrogen. The electrical resistance of the metal increases linearly with temperature, while that of the semiconductor decreases when temperature increases. In the case of the superconductor, below a transition temperature we observe a sharp reduction of the electrical resistance down to zero.

## §2 Overview

Measurement of the electrical resistance ($R$) is based on Ohm's law: $V = RI$, where the voltage ($V$) and the current intensity ($I$) follow a proportional relationship. The resistance of a material can thus be estimated by measuring the voltage across a sample while passing a known current intensity through it. In order to avoid the influence of resistance in conducting leads and contacts, we employ the four terminal method which is the standard for electrical resistance measurement. Additionally, the accuracy of measurements is influenced by the resolution of our measuring equipment. After getting an understanding of the method's principle, measurements of the electrical resistance are performed while bathing samples in ice or liquid nitrogen. Temperature of the sample is monitored with a thermocouple, and recorded automatically using a Arduino micro-controller. In parallel with these measurements, we also observe the Meissner effect in superconductors.

## §3 Measurement Principles

### 1. What is Electrical Resistivity?

Current in a material corresponds to the motion of charged particles such as free electrons. The equation for the motion of a single electron with charge $e$ subjected to an electrical field $\vec{E}$ is given as follows:

$$\vec{F} = e\vec{E} = m\frac{d\vec{v}}{dt} \tag{2.1}$$

We can solve this equation for the velocity $\vec{v}$:

$$\vec{v} = \frac{e\vec{E}}{m}t + \vec{v}_0 \tag{2.2}$$

Since electrons are continuously accelerated by the field $\vec{E}$, it follows that the current should also be continuously increasing. Or in the event that the electric field became zero, current should at least continue to flow. But in actual fact, electrons are scattered by impurities and lattice vibrations in the material. To take this scattering into account, the equation is corrected as follows:

$$e\vec{E} - m\frac{\vec{v}}{\tau} = m\frac{d\vec{v}}{dt} \tag{2.3}$$

When the electric field is zero, the velocity of electrons decays down to zero in accordance with the constant $\tau$. Moreover, electron velocity reaches a steady state when applying a constant electric field, such that $d\vec{v}/dt = 0$. Thus, the expression of electron velocity $\vec{v}$ in the steady state becomes:

$$\vec{v} = \frac{e\vec{E}}{m}\tau \tag{2.4}$$

For a volumetric density of electrons $n$, the current density $\vec{i}$ is:

$$\vec{i} = en\vec{v} = \frac{ne^2\tau}{m}\vec{E} \tag{2.5}$$

The current density $\vec{i}$ is proportional to the electric field $\vec{E}$, hence we can write:

$$\vec{i} = \sigma\vec{E} \tag{2.6}$$

$$\sigma = \frac{ne^2\tau}{m} \tag{2.7}$$

The coefficient $\sigma$ is called the "electrical conductivity".

$$\sigma = ne\mu \tag{2.8}$$

And $\mu = e\tau/m$ is called the mobility.

By defining the cross-sectional area of the sample $S$ and its length $L$, the current magnitude can be expressed as:

$$I = iS = ne\mu ES \tag{2.9}$$

By taking into account the electric field/potential relationship $E = V/L$, it follows that:

$$V = \frac{1}{ne\mu}\frac{L}{S}I \tag{2.10}$$

We can thus express:

$$R = \frac{1}{ne\mu}\frac{L}{S} \tag{2.11}$$

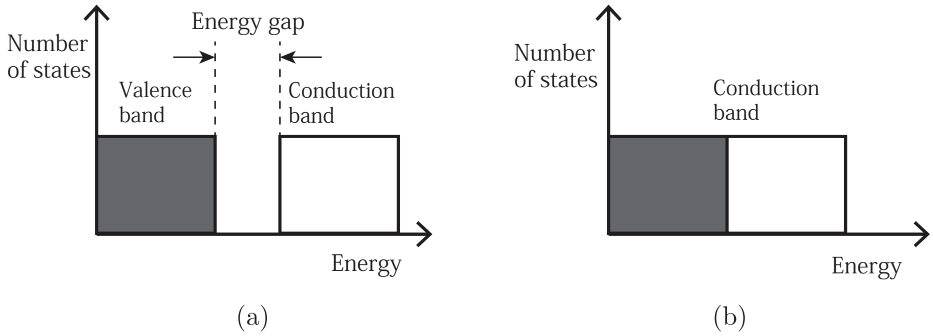



**Fig 2.1** Schematic Diagram of the Band Structure. Shaded portions indicate bands that are filled with electrons. (a) Band structure of semiconductors: it features an energy gap between a valence band filled with electrons and a conduction band not filled with electrons. (b) Band structure of metals: the conduction band is partially filled.

This allows us to write $V = RI$, which is Ohm's law.

Finally, the electrical resistivity is inversely proportional to conductivity:

$$\rho = \frac{1}{\sigma} \tag{2.12}$$

Electrical resistivity is a physical property of specific substances, which is determined experimentally by measuring both the shape and electrical resistance $R$ of samples.

## 2. Resistivity Dependence on Temperature

Electrical resistivity is a physical quantity that spans a wide range of values from $10^{-6}$ to $10^{18}$ Ohm cm, depending on the substance. Metals, semiconductors, and insulators can only roughly be identified by the value of their electrical resistivity. However, it is possible to clearly distinguish them by looking at changes in temperature.

In the case of insulators, the valence band is completely filled by electrons but a wide energy gap exists to the conduction band (Fig. 2.1(a)).

In the case of semiconductors, this energy gap is small. Electrons in the completely filled valence band do not contribute to the current when a voltage is applied. Only holes in the valence band contribute to the current, as a result of electrons becoming thermally excited into the conduction band (such semiconductor may also be called a Wilson semiconductor). The number of electrons excited from the valence band to the conduction band is:

$$n \propto \exp(-\frac{Q}{kT}) \tag{2.13}$$

Where $Q$ is the activation energy of the material, and $k$ is the Boltzmann constant:

$$k = 8.62 \times 10^{-5}\text{eV/K} = 1.38 \times 10^{-23}\text{J/K} \tag{2.14}$$

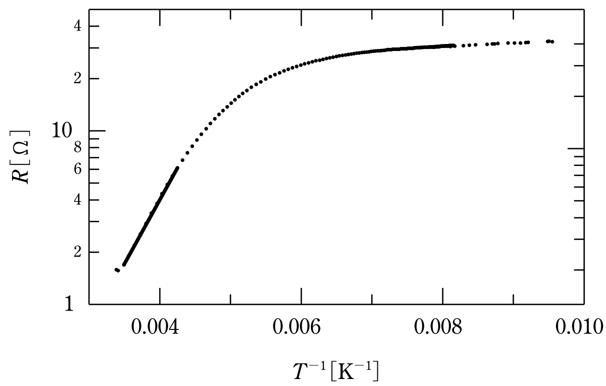



**Fig 2.2** Example of Semiconductor Resistance vs. Inverse of Temperature

(eV is a unit called electron volts). $T$ is the absolute temperature (unit K), which is related to the Celsius temperature $t$ by:

$$T = t + 273.15 \tag{2.15}$$

As the temperature rises, it can be seen from the equation above that the number of conduction electron $n$ increases, and thus the electrical conductivity $\sigma$ also increases. The temperature dependence of the electrical resistance is:

$$R \propto \frac{1}{\sigma} \propto \frac{1}{n} \propto \exp(\frac{Q}{kT}) \tag{2.16}$$

And taking the logarithm on both sides:

$$\log_{10} R = (\log_{10} e) \times \frac{Q}{kT} + constant \quad = 0.4343 \times \frac{Q}{kT} + constant \tag{2.17}$$

Therefore, a plot of $R$ as a function of $1/T$ contains a straight line portion as shown in Fig. 2.2. The activation energy $Q$ is derived from the slope of this straight line. In other words, the ratio of resistance between two points A and B can be expressed as $R_A/R_B = 10/1$.

$$Q = \frac{k}{0.434} \frac{1}{\frac{1}{T_A} - \frac{1}{T_B}} (\text{eV}) \tag{2.18}$$

Next, the activation energy of the semiconductor sample can be obtained (see textbook p.93, refer to the semi-logarithmic graph). The energy gap $E_g$ of the semiconductor is found to be twice the activation energy:

$$E_g = 2Q \tag{2.19}$$

At lower temperatures, the behavior deviates from a straight line and cannot be explained by the model above. Electrical resistivity at such temperatures is affected by impurities, and it is thus called the "impurity region".

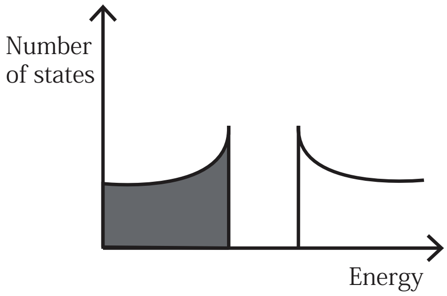



**Fig 2.3** Schematic Diagram of the Band Structure for Superconductors. Shaded portions indicate bands that are filled with electrons. It looks like an insulator in the sense that there exists an energy gap, but in actual fact they are totally different. The shaded portion corresponds to the superconducting state, and the unshaded portion indicates the ordinary conducting state (normal conducting state).

Finally, in the case of metals the conduction band is partially occupied by electrons (Fig. 2.1(b)). Therefore, the number of conduction band electrons is not affected much by temperature. The effect of temperature variations on resistivity is mainly due to the scattering of electrons by vibration of the constituent atoms of the material, so called "lattice vibration". Since lattice vibration becomes more violent with rising temperatures, the electrical resistance increases along with the temperature. At high temperatures, the electrical resistance is known as a result of theoretical calculations to be proportional to the temperature.

It is thus possible to understand the difference in electrical resistivity in various materials through their respective band structures. The key differences between semiconductors/insulators and metals are their band structures and whether electrons are located in the conduction band (we should mention the case of Mott insulators, not covered by the conventional band theory).

For some metals, below a certain temperature the electrical resistance becomes 0. This phenomenon is called superconductivity, and such materials are referred as superconductors. Fig. 2.3 shows that an energy gap exist between the normal state and superconducting state. The phase transition occurs at a certain critical temperature ($T_C$), below which neighboring electrons form a tie called "Cooper pairs". The characteristics of this natural phenomenon can be described as follows:

- The electrical resistance becomes zero.
- Magnetic fields cannot penetrate inside the superconductor (Meissner effect).
- The thermoelectric power becomes zero.

In 1986, Bednorz and Müller discovered the substance $La_{2-x}Ba_xCuO_4$ (x $\sim$ 0.15) which is superconducting at the critical temperature $T_C$ of 35 K. Since then, synthetic substances have been discovered one after another that exhibit superconductivity at ever higher $T_C$. Nowadays, several substances are known to have a $T_C$ above the temperature of liquid nitrogen (77.35 K).

**3. 2 Terminals vs. 4 Terminals**

There are two commonly used methods for measuring the electrical resistance of a sample. In the 2 terminals method, a current source sends an electrical current through contact leads connected to the sample. A voltmeter is connected in parallel with the current source through the same contact leads, and senses the voltage drop across the 2 terminals. While only 2 contact leads are required in this method, the measured voltage drop is due to the combined resistance of the sample and contact leads as follows:

$$V_{2T} = (R_{contact1} + R_{sample} + R_{contact2})I_{source} \tag{2.20}$$

When measuring samples with very small resistance (such as superconductors), the contact resistance can dominate the voltage drop. In such cases, the 4 terminals method provides much more sensitive measurements of the electrical resistance, by adding two more contact leads. The voltmeter, which has a very high internal resistance (several MΩ), draws a very small sensing current through these extra contact leads. The measured voltage drop is thus due to the combined resistance of the sample and contact leads as follows:

$$V_{4T} = (R_{contact1} + R_{contact2})I_{sense} + R_{sample}(I_{sense} + I_{source}) \tag{2.21}$$

Since the sensing current is very small compared to the source current, then the voltage drop across the contact leads becomes insignificant in comparison with the voltage drop across the sample. Then the relationship may be approximated as follows:

$$V_{4T} \approx R_{sample}I_{source} \tag{2.22}$$

All experiments described in this chapter are conducted using the 4 terminals method.

## §4 Experimental Apparatus

1. Arduino programmable micro-controller

   The Arduino is an open-source micro-controller board designed for experimental prototyping. It features 6 analog inputs and 13 digital I/O channels for expansion modules. The internal reference voltage of the Arduino is 5V. The 10 bit analog digital

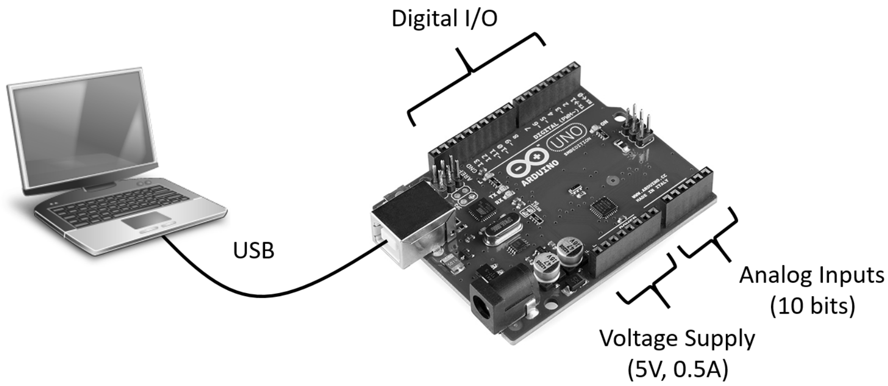



**Fig 2.4** Overview of Arduino and PC System

converters (ADC) can read voltages from 0 to 5V in 1024 steps.

2. Personal computer （Windows 8.1 or MacOSX）

   Used to program the Arduino and expansion modules in the various experiments.

3. Constant current power supply （ADVANTEST　R6144）

   Used to pass a constant current between terminals of the samples.

4. Resistor

   You will use this resistor to compare a 10 bit vs. 24 bit ADC, and verify Ohm's law.

5. K-Type Thermocouple

   This thermocouple can measure temperature between -200 and +1350 ° C.

6. Copper wire

   The copper wire has been wound to produce a small coil, which we will use to measure the resistivity of copper.

7. Small Dewar bottle

   It will be filled with ice, in order to lower temperature of the copper coil.

8. Samples and holder

   Samples include a semiconductor (Te) and superconductor ($Bi_{1.6}Pb_{0.4}Sr_2Ca_2Cu_{2.8}O_x$).

9. Large Dewar bottle

   It will be filled with liquid nitrogen, in order to lower temperature of the sample holder. If needed, Nitrogen can be topped up using the pouring bottle and a funnel.

## §5 Experiment （1） Ohmic Test

To measure electrical resistance, we use the programmable Arduino board. This device has 6 analog inputs, which are converted internally to digital values by a 10 bit analog/digital

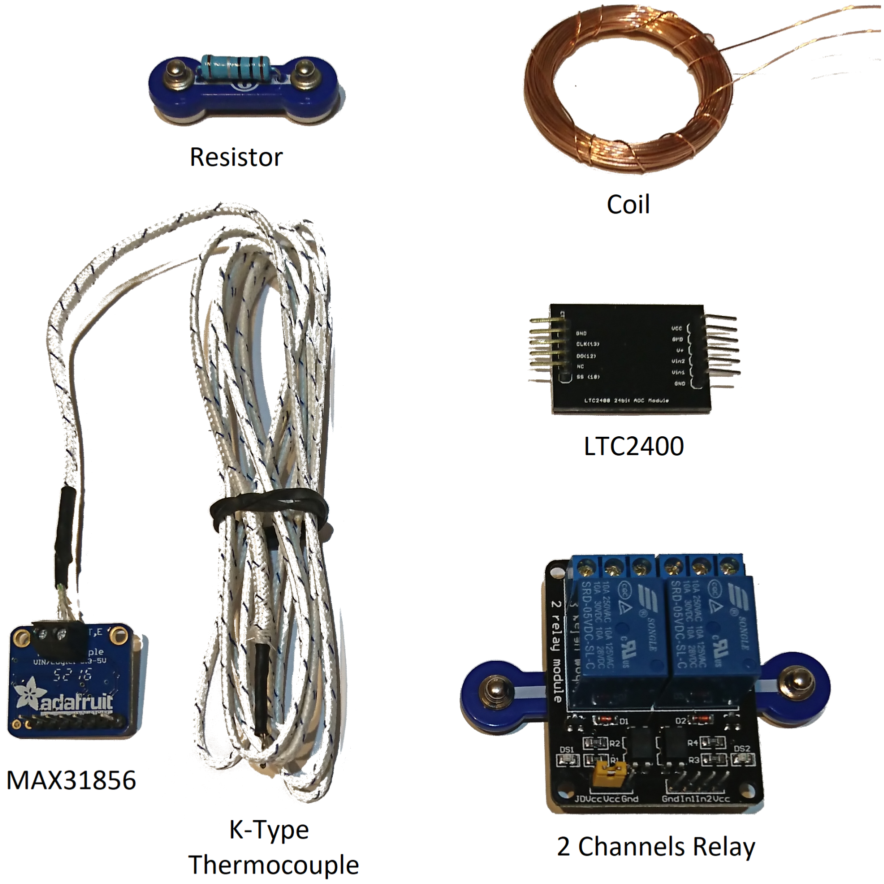



**Fig 2.5** Samples and Expansion Modules

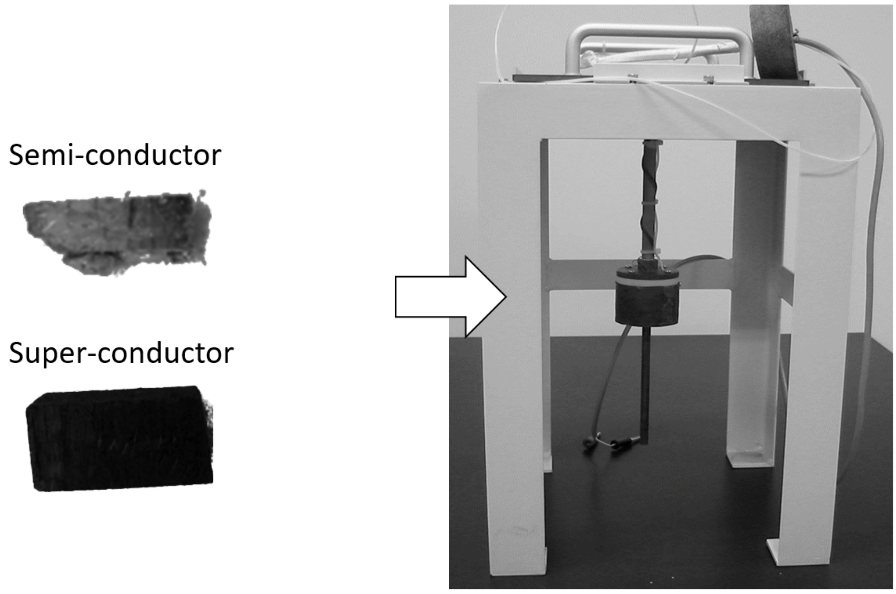



**Fig 2.6** Overview of Samples and Holder

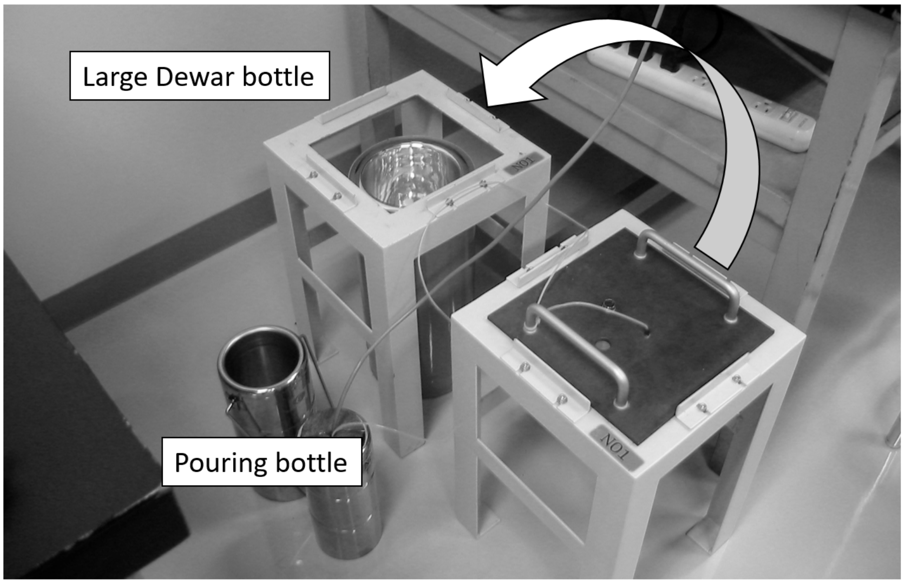



**Fig 2.7** Usage of Dewar Bottle and Samples Holder

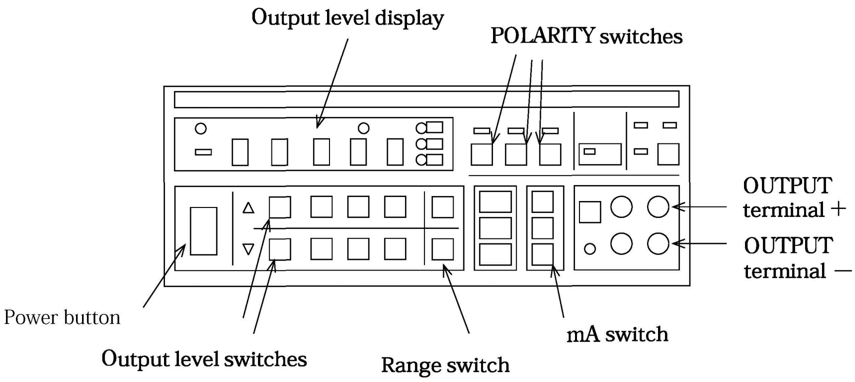



**Fig 2.8** Front Panel of Constant Current Supply R6144

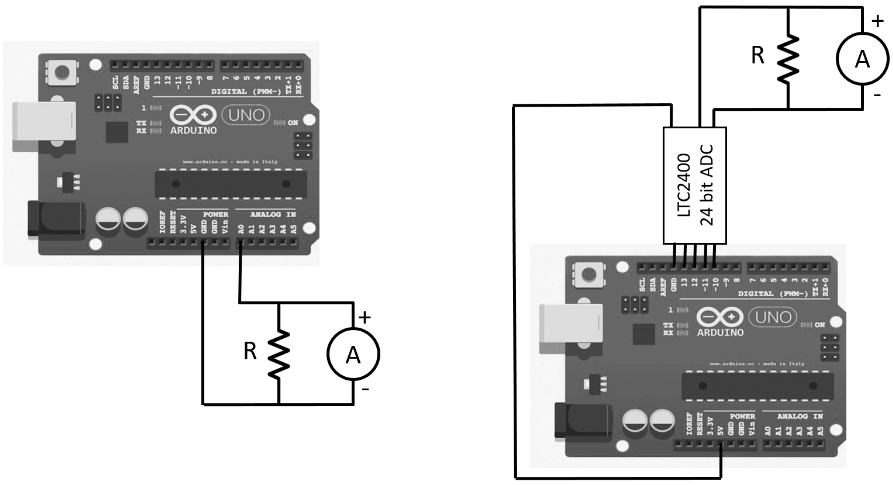



**Fig 2.9** Wiring Diagram for Ohmic Test

converter (ADC). When measuring small voltage drops, the sensitivity of the Arduino may be inadequate, so we will compare resistance measurement between the 10 bit Arduino and a 24 bit ADC expansion module.

## 1. Experimental Procedure

Measure the electrical resistance by recording voltage drop across a resistor, while increasing step-by-step the current flowing through the sample from the power supply.

### 1) Internal 10 bit ADC

1. Connect the constant current supply (ADVANTEST R6144) and Arduino to the unknown resistor as shown in Fig. 2.9 (left). The Arduino analog input 0 will be used to measure voltage drop in this experiment.
2. On the personal computer, open the Arduino application and enter this C code in the editor:

```
float Vref = 5.0;
float Vstp = 1024;

void setup() {
        // Initialize COM port
        Serial.begin(9600);
}

void loop() {
        // Display the measured voltage
        float V = analogRead(0) * Vref / Vstp;
        Serial.println(V,6);

        // Wait a little
        delay(1000);
}
```

3. After compiling and uploading the program to the Arduino board, start the serial monitor (Tools Menu, Serial Monitor).
4. Turn ON the R6144 constant current supply, and ensure that the OPERATE button is not illuminated.
5. Press the mA button on the front panel, then set output current to 1 mA.

6. Push the + POLARITY button, to set the current direction.
7. Push the OPERATE button, the current will start flowing.
8. Record voltage readings from the Arduino monitor while stepping current from 1 to 30 mA in steps of 1 mA (use spreadsheet software, installed on the PC).
9. Press the OPERATE button once more to stop the current flow, then unplug the USB cable from the computer to power off the Arduino, and close the monitor window.

**2) External 24 bit ADC**

1. Connect the R6144 constant current supply and Arduino to the unknown resistor as shown in Fig. 2.9 (right). The LTC2400 external Arduino module will be used to measure voltage drop in this experiment. On the LTC2400: pin VCC connects to the Arduino's 5V output pin, while pins Vin1 (positive) and GND (negative) connect to the resistor.
2. On the personal computer, open the Arduino application and enter this C code in the editor:

```
#include "LTC2400.h"

float Vref = 4.094;
float Vstp = 16777216;
LTC2400 ADC24bit = LTC2400(10,11,12,13,14);

void setup() {
        // Initialize COM port
        Serial.begin(9600);
}

void loop() {
        // Display the measured voltage
        float V = ADC24bit.Read(1) * Vref / Vstp;
        Serial.println(V,6);

        // Wait a little
        delay(1000);
}
```

3. After compiling and uploading the program onto the Arduino board, start the serial

connection monitor and repeat the same steps as the previous section to determine voltage drop while stepping current from 1 to 30 mA in steps of 1 mA. NOTE: after stepping the voltage, wait a few seconds and uncheck the "autoscroll" box. This will allow you to select several values, which you can enter in the spreadsheet and average automatically.

4. After completing the previous task, push the - polarity button to quickly inverse the current flow to -30 mA. Record the value of voltage only for -30 mA (you do not need to step negative voltages).
5. Press the OPERATE button once more to stop the current flow, then unplug the USB cable from the computer to power off the Arduino, and close the monitor window.

### 2. Analysis Tasks for Report

1. ADC resolution has a critical impact on our ability to measure the electrical resistance accurately. Consider how an ADC converts analog signals to digital values, and explain why the standard Arduino is not adequate for measuring small resistance values.
2. Using the color code poster on the wall of the laboratory room, determine the specification resistance of the sample used in this experiment. Compare with the experimentally measured value.
3. What is thermoelectric power, and why is it possible to determine it by inverting the current polarity? Please determine the thermoelectric power from your measurements.

## §6 Experiment (2) Resistivity of Copper

In this experiment, we will use a small copper coil of total length 10m and diameter 0.4mm to measure the variation in electrical resistance between room temperature and 5 °C. Temperature will be measured by a thermocouple connected to a MAX31855 amplifier. This Arduino expansion module contains an internal temperature sensor, for cold-junction compensation.

### 1. Experimental Procedure

1. Connect the coil and thermocouple to the Arduino and constant current supply as shown in Fig. 2.10.
2. On the personal computer, open the Arduino application and enter this C code in the editor:

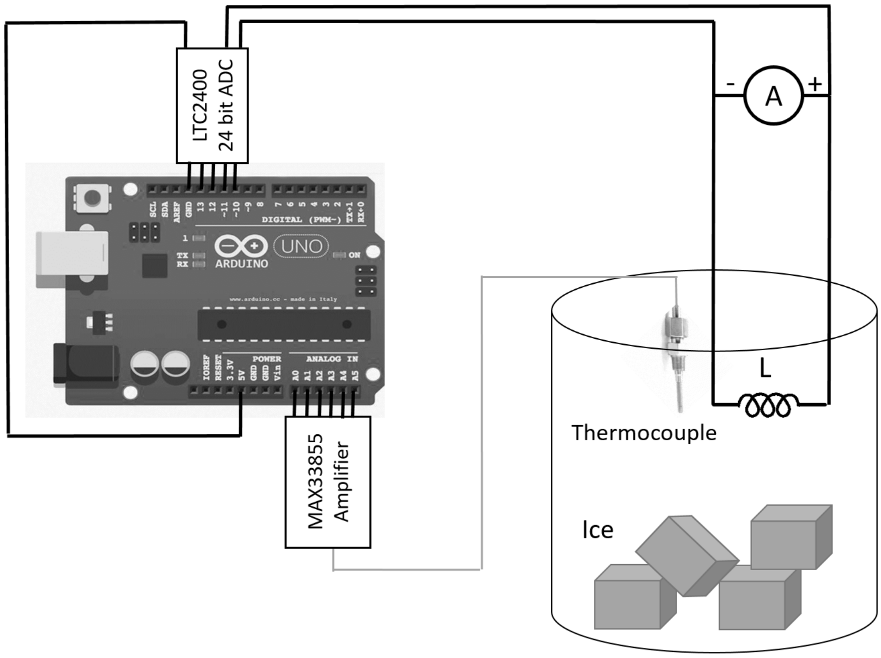



**Fig 2.10** Wiring Diagram for Copper Resistivity Test

```
#include "LTC2400.h"
#include "MAX31855.h"

float Vref = 4.094;
float Vstp = 16777216;
LTC2400 ADC24bit = LTC2400(10,11,12,13,14);
MAX31855 Thermocouple = MAX31855(A0,A1,A2,A3,A4,A5);

void setup() {
        // Initialize COM port
        Serial.begin(9600);
    Serial.println("Temperature (C), Voltage (V)");
}

void loop() {
        // Display the measured temperature
        float T = Thermocouple.Read();
```

```
        Serial.print(T,2); Serial.print(",");

        // Display the measured voltage
        float V = ADC24bit.Read(1) * Vref / Vstp;
        Serial.println(V,6);

        // Wait a little
        delay(1000);
}
```

3. After compiling and uploading the program onto the Arduino board, start the serial connection monitor and set current on the R6144 to 100 mA.
4. Push the OPERATE button to start the current flow.
5. Insert ice into the lower half of the small Dewar flask, then insert the thermocouple and coil and cover with the cork cap.
6. When temperature becomes close to 5, press the OPERATE button to stop current flow.
7. Unplug the arduino's USB cable, then copy/paste contents of the serial monitor to a spreadsheet and save the file.

**2. Analysis Tasks for Report**

1. Use the electrical resistance of copper at each temperature together with the dimensions of the wire (section area $a$, and length $l$) to derive resistivity from the following formula:

$$\rho = R\frac{a}{l}$$

2. Graph the relationship between the calculated resistivity and temperature. Obtain the magnitude of resistivity near 0 C° and the slope of resistivity with respect to temperature $d\rho/dT$, and compare them with reference values from the literature. (At 0 C°, the resistivity is $1.55\times10^{-8}\Omega\cdot$ m, and the slope with respect to temperature is $6.87\times10^{-11}\Omega\cdot$ m /K.)

## §7 Experiment (3) Resistivity of Semi and Super-Conductor

We measure the variation in electrical resistance with temperature for a semiconductor and superconductor, down to the temperature of liquid nitrogen. The temperature and voltage measurements will be taken automatically by the Arduino. The voltage drop cannot

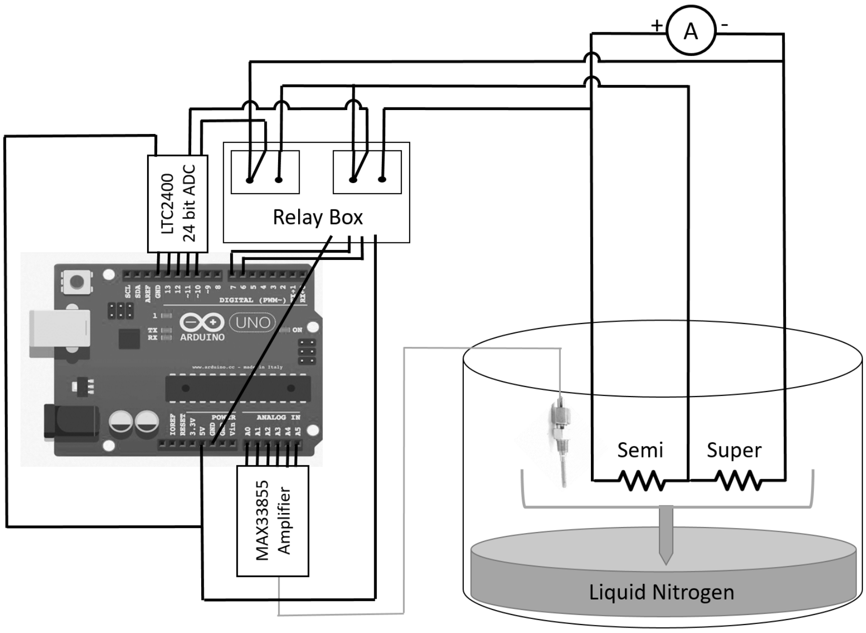



**Fig 2.11** Wiring Diagram for Semi and Super-Conductor Resistivity Test

be measured across both samples simultaneously, because grounding would occur on the Arduino. In order to measure each sample individually, we will add a relay box module to our Arduino setup.

The relay box module is powered by the Arduino's 5V/GND pins (GND and VCC on the relay module) and controlled by the Arduino's digital pins 6 and 7 (In1 and In2 on the relay module). The code below shows the principle of operation:

```
void setup() {
        pinMode(6,OUTPUT);
        pinMode(7,OUTPUT);
}
void loop() {
        // Wait -> Switch relays to channel 0 -> Wait
        delay(1000);
        digitalWrite(6,0);
        digitalWrite(7,0);
        delay(1000);

        // Wait -> Switch relays to channel 1 -> Wait
```

```
        delay(1000);
        digitalWrite(6,1);
        digitalWrite(7,1);
        delay(1000);
}
```

**1. Experimental Procedure**

1. Connect the samples and thermocouple to the Arduino and constant current supply as shown in Fig. 2.11. Use the color coded cables to ensure that the setup is correctly connected.
2. On the personal computer, open the Arduino application and write by yourself the code necessary to operate the 24 bit ADC, thermocouple, and relay box.
3. After compiling and uploading the program onto the Arduino board, start the serial connection monitor and set current on the R6144 to 100 mA.
4. Fill the large Dewar flask with liquid nitrogen, up to the black mark.
5. Push the OPERATE button to start the current flow, then set the sample holder in the large Dewar flask containing the liquid nitrogen.
6. When the temperature reaches roughly 100 - 110 K, the drop in temperature becomes very slow. At that time, top up the liquid nitrogen little by little using a funnel, until measurement of the superconducting transition is completed.
7. Press the OPERATE button to stop current flow, disconnect the Arduino's USB cable, then copy/paste contents of the serial monitor to a spreadsheet.

**2. Analysis Tasks for Report**

1. Represent the relationship between the electrical resistance of the semiconductor and $1/T$ on semi-log graph paper and confirm that it has a straight line part from the room temperature to the low-temperature side. Estimate the activation energy from the slope of this line to obtain the energy gap, and compare it with the reference value from the literature.
2. Graph the temperature change of the electrical resistance of the superconductor and determine the critical temperature for superconductivity from the mean value of the temperature where the electrical resistance suddenly begins to decrease and the temperature where the electrical resistance becomes zero.

# §8 Appendix

## 1. Electrical Resistance of Metals

| Substance Name | Resistivity at 0 C° ($10^{-8}$ Ω· m) |
|---|---|
| Al | 2.50 |
| Sb | 39 |
| Au | 2.05 |
| Ag | 1.47 |
| Cu | 1.55 |
| In | 8.0 |
| Pb | 19.2 |

## 2. Energy Gap of Semiconductors

| Substance Name | Energy Gap (eV) |
|---|---|
| Ge | 0.785 |
| Si | 1.206 |
| GaAs | 1.53 |
| InSb | 0.17 |
| ZnSe | 2.83 |
| CdS | 2.582 |
| Te | 0.33 |

### 3. Critical Temperatures of Superconductors

| Substance Name | Critical Temperature (K) |
|---|---|
| Al | 1.196 |
| In | 3.4035 |
| Pb | 7.193 |
| Sn | 3.722 |
| $MgB_2$ | 39 |
| $Nb_3Sn$ | 18.3 |
| $PuCoGa_5$ | 18.5 |
| $YBa_2Cu_3O_7$ | 90 |
| $Bi_2Sr_2Ca_2Cu_3O_{10+\delta}$ | 110 |
| $TlBa_2Ca_2Cu_3O_x$ | 133.5 |
| $\kappa$ - $(BEDT\text{-}TTF)_2Cu(NCS)_2$ | 10.4 |
| $(TTF)[Pd(dmit)_2]_2$ | 6 (19 kbar) |
| $C_{60}RbCs_2$ | 33 |

# Experiment 3. Measurement of the Magnetic Field of a Coil using a Hall Element

## §1 Experimental Objective

It is well known that the magnetic field $H$ inside an infinitely long coil relates to the electric current $I$ and the number of coils in a unit length $n$ by the formula $H = nl$. The units here are [A/m], [A] and [coils/m] for $H$, $I$ and $n$ respectively. It is curious why the shape of the coil, such as its radius and total length, does not matter at all in this simple formula. We know that an infinitely long coil does not exist, and there must be two opposite ends for any coil (unless the coil is rounded up like a torus by joining the two ends together). It is hence natural to wonder if this formula really applies accurately to coils in real life or not. This experiment will let you answer this question yourself. In the process, you will also learn about how to use a Hall Element to measure magnetic fields, the underlying concept of Hall Effect, and the properties of the magnetic field created by a steady electric current.

## §2 Overview

- **Experiment 1.** To understand the Hall Effect, the underlying mechanism of a Hall Element, you are going to perform the following measurements: (1) Measure the Hall voltage as you vary the Hall current under a fixed magnetic field strength ($H = 20000$ [A/m]), to understand the interdependence between the Hall voltage and current; (2) Measure the Hall voltage as you vary the magnetic field strength under a fixed Hall current (10 [mA]), to understand the interdependence between the Hall voltage and magnetic field.
- **Experiment 2.** You will investigate the spatial distribution of the magnetic field created by a coil using the Hall Element you have characterized in Experiment 1.
- **Experiment 3.** You will repeat Experiment 2 for coils of different lengths and radii to investigate the impact of the coil shape on the magnetic field configuration.

## §3 Principles

### The Ampère's Law

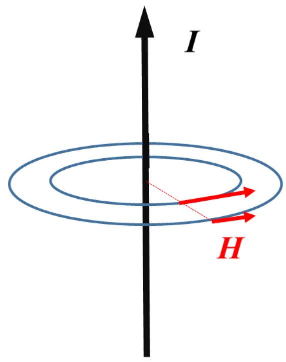



**Fig 3.1** Magnetic field configuration of a steady current along a straight wire

As an electric current flows along a straight wire, a circular magnetic field centering on the wire is generated. The magnetic field direction is anti-clockwise as the current flow is pointed up (see Figure 3.1). The contour integral of this magnetic field $H$ along any closed loop around the current $I$ has a non-zero finite value. This value does not depend on the exact shape of the closed loop, but only on the total current flowing inside this loop. In other words, for any closed loop $C$,

$$\oint_C \boldsymbol{H} d\boldsymbol{s} = NI$$

where $NI$ is simply the total current flowing in the space enclosed by $C$. Note that $NI$ does not include any current outside the loop $C$. This relation is known as the Ampère's Law (see Figure 3.1).

Using the Ampère's Law, let's consider the magnetic field $H(r)$ at a radius $r$ from a straight wire with steady current $I$. The magnetic field created by a current along a straight wire is distributed on a plane perpendicular to the current direction with a configuration of concentric circles. Its strength is determined by the radial distance $r$ from the wire and the current $I$. Now, if we apply the Ampère's Law on a circle of radius $r$ centered on the straight wire, the distance to the wire for any given point on this circle is always the same, $|\boldsymbol{H}|(= H)$ is hence constant, so that

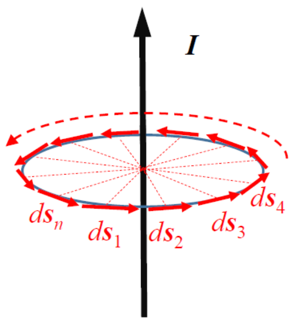



**Fig 3.2** A contour integration along a loop surrounding a straight line current $I$ when applying the Ampère's Law

$$\begin{aligned}\oint_C \boldsymbol{H} d\boldsymbol{s} &= H ds_1 + H ds_2 + H ds_3 + \cdots + H ds_n \\ &= H(ds_1 + ds_2 + ds_3 + \cdots + ds_n) \\ &= H \times 2\pi r \\ &= I \qquad \text{(see Figure 3.2)}\end{aligned}$$

As a result, the magnetic field $H(r)$ on a circle with radius $r$ centered on the wire is

$$H(r) = \frac{I}{2\pi r}$$

**The biot-Savart's Law**

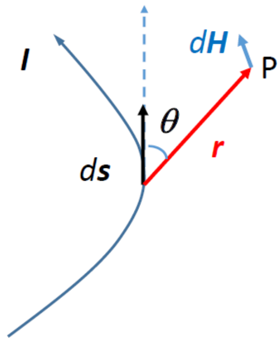



**Fig 3.3** Layout diagram for the biot-Savart's Law

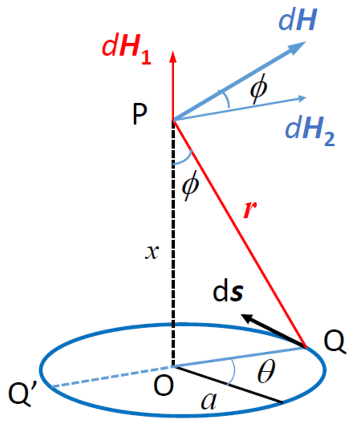



**Fig 3.4** The magnetic field at point P created by a circular current

Consider an infinitesimal segment $d\boldsymbol{s}$ of a wire with a steady current $I$. At a point P of displacement $\boldsymbol{r}$ from $d\boldsymbol{s}$, the magnetic field $d\boldsymbol{H}$ generated by this segment is

$$d\boldsymbol{H} = \frac{I}{4\pi}\frac{d\boldsymbol{s} \times \boldsymbol{r}}{r^3}$$

If we define $\theta$ as the angle between $d\boldsymbol{s}$ and $\boldsymbol{r}$, then the magnetic field $dH = |d\boldsymbol{H}|$ can be rewritten as

$$dH = \frac{I}{4\pi}\frac{ds \sin\theta}{r^2}$$

, and its vector is perpendicular to the plane defined by $d\boldsymbol{s}$ and $\boldsymbol{r}$, pointing toward a direction according to the "right-hand rule" (i.e., if you wrap your right hand fingers from $d\boldsymbol{s}$ to $\boldsymbol{r}$, then $d\boldsymbol{H}$ is along the direction of your thumb, see Figure 3.3).

Using the biot-Savart's Law, we can now think about the magnetic field at a point P at a distance $x$ directly above the center of a circular current $I$ of radius $a$ (see Figure 3.4). The contribution $dH$ of an infinitesimal segment $d\boldsymbol{s}$ of the circular current at point Q to the magnetic field at point P can be calculated by realizing that $\theta = \pi/2$ and $r = \sqrt{a^2 + x^2}$, such that

$$dH = \frac{Ids \sin\theta}{4\pi r^2} = \frac{Ids}{4\pi(a^2 + x^2)}$$

Now, it is convenient to think about the magnetic field contribution from another point Q' directly opposite to Q on the circular current. The component $d\boldsymbol{H_2}$ of $d\boldsymbol{H}$ perpendicular to the central axis (dashed line joining O and P in Figure 3.4) from Q and Q' respectively are exactly opposite to each other, and they cancel out. Since this is always true for any pair

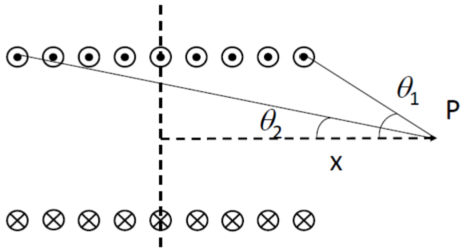



**Fig 3.5** Estimating the magnetic field of a solenoid coil at point P

of opposite points on the current, from symmetry, the magnetic field $\boldsymbol{H}$ at point P must be parallel to the central axis. It is hence sufficient to consider only the component $d\boldsymbol{H_1}$ parallel to the central axis, which from $dH_1 = dH \sin\phi$ and $\sin\phi = a/r$ can be calculated to be

$$dH_1 = \frac{Ids}{4\pi(a^2+x^2)}\frac{a}{r} = \frac{Iads}{4\pi(a^2+x^2)^{3/2}}$$

As a result, the total magnetic field generated by a circular current along the central axis can be readily estimated by rewriting $ds = ad\varphi$ and integrating over all segments, i.e.,

$$H_1 = \int dH_1 = \frac{Ia}{4\pi(a^2+x^2)^{3/2}}\int_0^{2\pi} ad\varphi = \frac{Ia^2}{2(a^2+x^2)^{3/2}}$$

In the case of $x = 0$, we can immediately reproduce the well-known result of the magnetic field strength at the center of a circular current,

$$H = \frac{I}{2a}$$

### The magnetic field of a solenoid coil of finite length

Using the magnetic field $H$ we estimated above for a circular current, we can now think about the case of a solenoid coil of finite length. Figure 3.5 shows a schematic diagram of a solenoid coil with radius $a$, length $2l$ and $n$ loops per unit length. For a point P on the central axis of the coil and at a distance $x$ from the coil center, the magnetic field can be written down as

$$\begin{aligned} H(x) &= \int_{x+l}^{x-l} \frac{nIa^2dx}{2(a^2+x^2)^{3/2}} = -\frac{nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta d\theta = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1) \\ &= \frac{nI}{2}\left[\frac{x+l}{\sqrt{a^2+(x+l)^2}} - \frac{x-l}{\sqrt{a^2+(x-l)^2}}\right] \end{aligned}$$

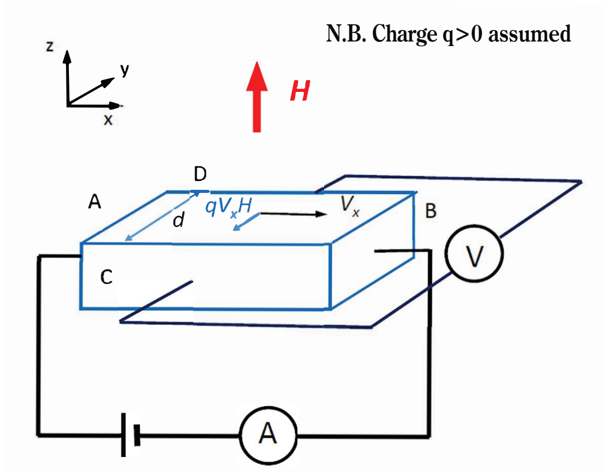



**Fig 3.6** Conceptual diagram of the Hall Effect

**Hall Effect**

In this experiment, we will use a Hall element to measure magnetic fields. As its name implies, a Hall element uses the Hall Effect to probe magnetic fields. It is hence instructional here to first introduce the concept of Hall Effect. If you are wondering, those popular Gaussmeters frequently used for magnetic field measurements are also basing their working principle on the Hall Effect.

It is well known that if a current is flowing inside a sample, and an external magnetic field is applied perpendicular to the current, an electric field is generated in the direction orthogonal to both the current and the magnetic field. The phenomenon was discovered by Edwin Herbert Hall after whom the effect was named (Hall Effect). The polarity and amplitude of the electric voltage (Hall voltage) generated by the magnetic field are correlated with the movement of charges inside the sample.

When a magnetic field $\boldsymbol{H}$ is experienced by a charge $q(>0)$ moving at a velocity $\boldsymbol{v}$, a force is exerted onto the charge and change its path. This force is known as the Lorentz force, which can be written down by the vector product

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$$

Again, by the right-hand rule, the force direction is orthogonal to the charge velocity and the magnetic field. Here, $\boldsymbol{B}$ is the magnetic flux density, which is related to the magnetic field by $\boldsymbol{B} = \mu_0 \boldsymbol{H}$ where $\mu_0 = 4\pi \times 10^{-7}\mathrm{N/A^2}$ is the magnetic permeability in vacuum. In a Hall element as illustrated in Figure 3.6, a steady current is flowing in the direction $x$ from A to B, and a uniform magnetic field is applied in the direction $z$. For a charge $q$ in the element that is moving at a velocity $v_x$, the resulted Lorentz force on the charge is in the $-y$ direction. The charge distribution in the $y$ direction is hence altered. This charge separation (between +ve and -ve charges) results into a static electric field $\boldsymbol{E}$. This field exerts a force on the charges which is exactly opposite and of the same magnitude as the Lorentz force, so that a steady state is achieved inside the Hall element. This balance of force in the $y$ direction can be written as

$$q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B} = 0$$

which can be separated into the components

$$qE_y - qv_xB_z = 0, \qquad E_y = v_xB_z$$

If the charge density is denoted by $n$, the current density in the element is simply $i_x = nqv_x$, so that

$$E_y = Ri_xB_z, \qquad R = 1/nq$$

The proportionality constant is called the "Hall coefficient", which is +ve if the majority of the charge carriers in the element has a positive charge, or -ve if most of them are electrons. The sign and magnitude of the Hall coefficient can be sensitive to temperature and other environmental factors. It is an important physical quantity that describes the electrical conduction properties of different materials.

Here, let the distance between C and D be $w$ and the thickness of the element be $d$, so that the total current is $I_{\mathrm{H}}[= (wd)i_x]$. The Hall voltage $V_{\mathrm{H}}$ (using point C as a reference) is then

$$V_{\mathrm{H}} = -E_yw = -R\frac{I_{\mathrm{H}}}{wd}B_zw = -\frac{RI_{\mathrm{H}}B_z}{d}$$

In other words, by calibrating the Hall voltage by applying known magnetic fields to the Hall element, one can measure unknown magnetic fields through measuring the Hall voltage.

## §4 Experimental Devices

Figure 3.7 shows the devices you will need in this experiment.

**Hall element and its current generator and voltmeter**

These are the devices you will use to measure the Hall voltage. As mentioned above, you will first calibrate the Hall element by measuring the voltage under different known magnetic fields, so that you can use the element for magnetic field measurement. The Hall element Toshiba THS118 which uses semiconductor GaAs is placed inside an acrylic cylinder (see Figure 3.8).

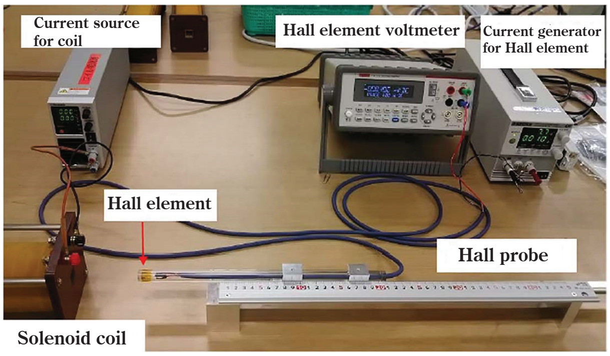



**Fig 3.7** Experimental setup


**Fig 3.8** The Hall element made of semiconductor THS18, GaAs by Toshiba

### Solenoid coil and its steady current source

The solenoid coil you will use has 6000 windings and a full length $2l = 0.3[m]$, so that the coil density $n$ is $n = 20000$ [1/m]. By varying the steady current in the coil, you can change the magnetic field strength. If time allows, you are encouraged to investigate the magnetic fields of solenoid coils of different shapes.

## §5 Experimental Procedures

### The relation between Hall voltage ($V_H$) and the applied magnetic field ($H$)

First, connect the current generator (for Hall element) and the voltmeter to the Hall element. Then place the Hall element at the center of the solenoid coil as illustrated in Figure 3.9. [Caution!!] The Hall element is super fragile. Please pay special attention as you handle it (do not ever apply a current higher than 10 [mA] to the element, or you will burn it).

- Turn on the voltmeter, with the filter switch on (i.e., press [shift] + [filter]).
- At first, under no magnetic field $H = 0$, increase the Hall current from 0 to 10[mA] in steps of 1[mA]. Measure the changing Hall voltage ($V_H$) every time, and plot a graph of Hall current ($I_H$) versus Hall voltage ($V_H$).

  Then, set the applied magnetic field to $H = 20000$ [A/m] (by keeping the current in the solenoid coil at 1 [A]), and repeat the measurements above. Again, plot the Hall current ($I_H$) against Hall voltage ($V_H$) in this case.

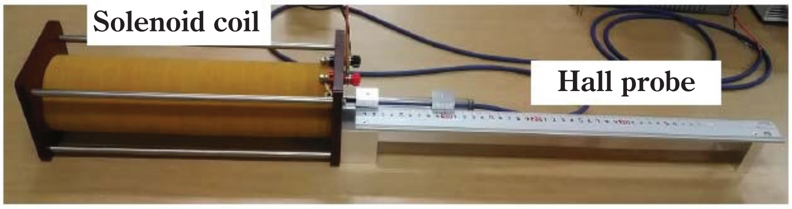



**Fig 3.9** Place the Hall element at the center of the coil.

- This time, set the Hall current to 10[mA], and turn off the current in the coil for once. You will see that the Hall voltage does not drop to zero. It is because of the ambient magnetic field in the lab (e.g., terrestrial magnetic field) and noise from the Hall element itself. In other words, this voltage value is the "background" for your measurements. Mark down this voltage. You should then press the [null] button of your voltmeter to eliminate (or to zero) this background voltage.
  Then, increase the coil current from 0 to 1 [A] in steps of 0.1 [A], and measure the changing Hall voltage every time. By converting the coil current to magnetic field, plot a graph of applied magnetic field ($H$) versus Hall voltage ($V_{\mathrm{H}}$) to study their relation.

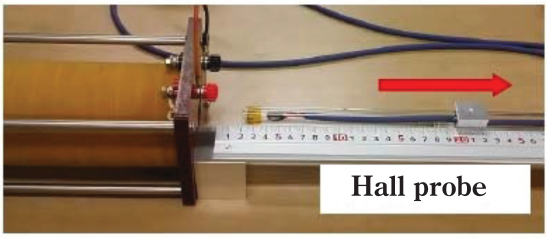



**Fig 3.10** Measuring the Hall voltage $V_{\mathrm{H}}(x)$ by moving the Hall element along the central axis of the coil in the $x$ direction.

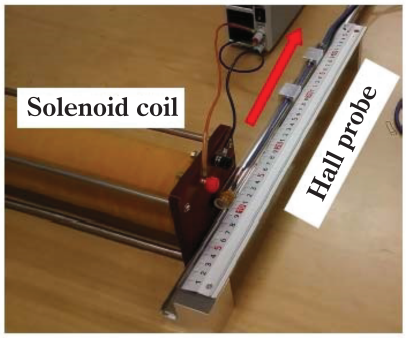



**Fig 3.11** Measuring the Hall voltage $V_{\mathrm{H}}(y)$ by moving the Hall element perpendicular to the central axis in the $y$ direction.

**Observation of the magnetic field configuration of a solenoid coil**

Set the Hall current to 10 [mA] and the coil current to 1 [A].

- First, place the Hall element at the center of the coil. Then, move the element along the central axis (see Figure 3.10). As you displace the element, measure the changing Hall voltage $V_{\mathrm{H}}(x)$ as a function of distance $x$ [m] from the coil center. You have to judge yourself how many points to take, but you will need to make finer measurements

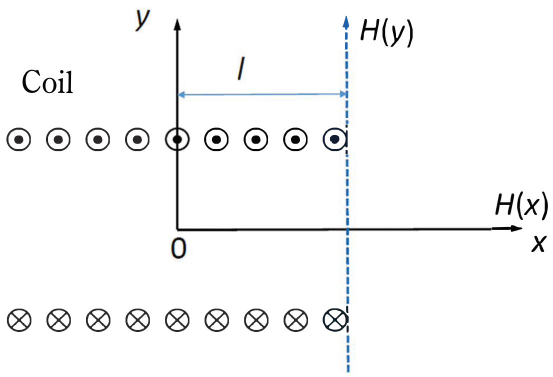



**Fig 3.12** The orientation of $H(x)$ and $H(y)$ relative to the coil

near the opening of the coil. Then, using the $V_H - H$ relation you have found above, estimate the magnetic field $H(x)$ as well.

- Near the coil opening, measure the Hall voltage $V_H(y)$ by moving the Hall element in the $y$ direction perpendicular to the coil's central axis (see Figure 3.11). Make sure you make finer measurements close to the opening. Then, similar to above, estimate the magnetic field $H(y)$ along the direction perpendicular to the coil opening. Here, becareful of the definition of direction $y$ in Figure 3.12. Look carefully at the current direction in the coil. You should also try to visualize the magnetic field configuration of the coil using a compass.
- (continuation) Repeat the measurement of $H(x)$ for a solenoid coil of a different shape.

## §6 Analysis and Discussion

- From the Ampère's Law, derive the equation $H(r) = I/2\pi r$ for the magnetic field at a distance $r$ from a direct current.

  Also, derive the equation

$$H(x) = \frac{nI}{2}\left[\frac{x+l}{\sqrt{a^2+(x+l)^2}} - \frac{x-l}{\sqrt{a^2+(x-l)^2}}\right] \tag{3.1}$$

  for the magnetic field along the central axis of a solenoid coil.
- For a Hall element, plot the graphs $I_H - V_H$ and $H - V_H$ to figure out their inter-relations. Also, from the graph, derive the equation of $V_H$(mV) as a function of $H$(A/m). Can you tell how big the magnetic field generated by a solenoid coil is compared to the background terrestrial magnetic field?
- Let $H(x)$ be the magnetic field at a point P at a distance $x$ from the coil center. Plot

the graph of $[x,\ H(x)]$. Compare the field strength between the center and near the coil opening. Discuss about this experimental result using equation 3.1. Also, is the $H(x)$ you measured consistent with equation 3.1?

Let $H(y)$ be the magnetic field along the $y$ direction as defined in Figure 3.12 (let $y = 0$ at the coil center). Plot the graph of $[y,\ H(y)]$. How does $H(y)$ behave near $y = 0$?

- Let $H(x_i)$ be the magnetic field at point $P_i$:$x_i$ (i.e., where you took the data points as you measured $H(x)$ above), we can define

$$\Delta H(x_i) \equiv \left[ \frac{x_i + x_{i+1}}{2}, \frac{H(x_{i+1}) - H(x_i)}{x_{i+1} - x_i} \right]$$

to investigate how $H$ varies with $x$. Plot the graph of $\Delta H(x)$ and $\Delta H(y)$. Discuss the difference between the coil center and near the coil opening in your graph using equation 3.1. Find out the range of the regions where the magnetic field varies with $x$ and $y$.

At the point $(x, y) = (0.175\text{m}, 0)$, use the $\Delta H(x)$ and $\Delta H(y)$ above to roughly estimate the quantity div$\boldsymbol{H}$ as follows.

$$\text{div}\boldsymbol{H} = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z}$$

Refer to the Appendix to learn about the meaning of this quantity. Here, as an approximation, you can substitute the partial differentials in this equation by the differences above. Also, from the $yz$ symmetry of the coil, you can rewrite the equation as

$$\text{div}\boldsymbol{H} = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} \sim \frac{\partial H_x}{\partial x} + 2\frac{\partial H_y}{\partial y}$$

- By applying the Ampère's Law to a solenoid coil, i.e.,

$$\oint_C \boldsymbol{H}_s d\boldsymbol{s} = NI,$$

discuss about $N$ (the total number of windings of the coil) and the magnetic field strength.

- (continuation) Sketch the magnetic field configurations of solenoid coils of different shapes. Also, use the Ampère's Law to derive their respective $N$.

**Appendix**

- On the unit of magnetic field,

In the SI unit system, the unit of magnetic field is $H$ (A/m), while in the cgs system it is $H$ (Oe: Oersted). However, in the cgs system, the magnetic flux density is $B$ (G:

Gauss), and in a vacuum 1 (Oe) = 1 (G), so it is common to simply use (G: Gauss) as the basic unit of magnetic field. The conversion between the two systems is 1(Oe), (G) = 79.6 (A/m).

- We can write the Ampère's Law $\oint_C \boldsymbol{H} d\boldsymbol{s} = I$ using the vector manipulation symbol rot (rotation) as

$$\mathrm{rot}\boldsymbol{H} = \boldsymbol{I}$$

Here, the definition of rot $\boldsymbol{H}$ is

$$\mathrm{rot}\boldsymbol{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\boldsymbol{e}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\boldsymbol{e}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\boldsymbol{e}_z$$

where $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ are the unit vectors in the $x, y, z$ directions respectively.

- On the vector manipulation symbol div (divergence),
Let $\boldsymbol{V}(x, y, z)$ be any vector function, then div $\boldsymbol{V}$ is defined as

$$\mathrm{div}\boldsymbol{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

which basically describes the "outward flux" of the field $\boldsymbol{V}$ from any point $(x, y, z)$. Note that div$\boldsymbol{V}$ is a scalar, no longer a vector. Let's think a little more about its meaning here.

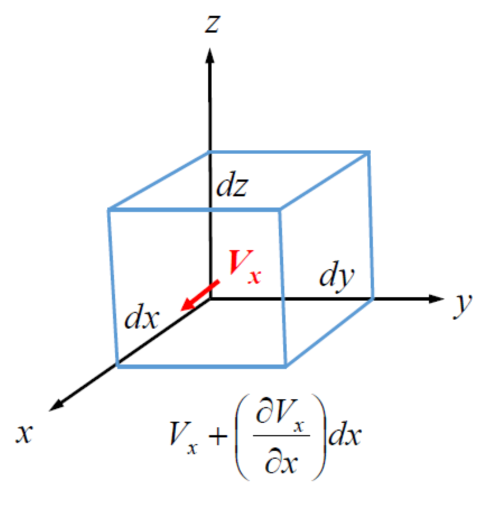



**Fig 3.13** Schematic diagram showing the flux of a vector field $\boldsymbol{V}$ in the $x$-direction.

Consider an infinitesimal volume with six faces defined by ($dx$, $dy$, $dz$) from the origin (Figure 3.13), and the number of streamlines crossing its faces. The number of streamlines flowing into the face $x = 0$ perpendicular to $x$ is simply the $x$-component of $\boldsymbol{V}$ multiplied by the face area $dydz$, i.e., $V_x dydz$. On the opposite side, we have the face $x = dx$, and the $x$-component of $\boldsymbol{V}$ there is $V_x + \frac{\partial V_x}{\partial x} dx$, so that the number of streamlines flowing out of the face $x = dx$ is

$$\left( V_x + \frac{\partial V_x}{\partial x} dx \right) dydz$$

As a result, the net number of streamlines flowing out of the faces perpendicular to $x$ is

$$\frac{\partial V_x}{\partial x} dxdydz$$

Similarly, for the faces perpendicular to $y$, it is

$$\frac{\partial V_y}{\partial y} dxdydz$$

, and for the faces perpendicular to $z$, it is

$$\frac{\partial V_z}{\partial z} dxdydz$$

The total number of streamlines flowing out of this infinitesimal volume is then the sum of these three,

$$\mathrm{div}\boldsymbol{V} dxdydz$$

Now, $dxdydz$ is just the volume of this six-faced cube, so we can understand div$\boldsymbol{V}$ as the number of streamlines flowing out of a unit volume at any given point in the field.

- For any magnetic field $\boldsymbol{H}$,

$$\mathrm{div}\boldsymbol{H} = 0 \qquad \text{(A)}$$

is believed to be always true in nature. It means that the net number of magnetic field lines flowing out of any unit volume must be zero! As one can see in Figure 3.14, field lines emerging from $+m$ must return to $-m$. In this situation, it is easy to see that the net number of field lines crossing the spherical volume denoted by the red dashed line must be zero. We say that, a magnetic monopole does not exist in the Universe.

The story is completely different for electric fields. As shown in Figure 3.15, the electric field line from a charge $+Q$ is diverging in the radial direction. In this case, the net number of electric field lines crossing a spherical volume is of course non-zero. Actually, if we write down the divergence of $\boldsymbol{E}$, it is

$$\mathrm{div}\boldsymbol{E} = \frac{Q}{\varepsilon_0} \qquad (\varepsilon_0 : \text{dielectric constant in vacuum}) \qquad \text{(B)}$$

So unlike the non-existence of magnetic monopoles, electric monopoles are everywhere.

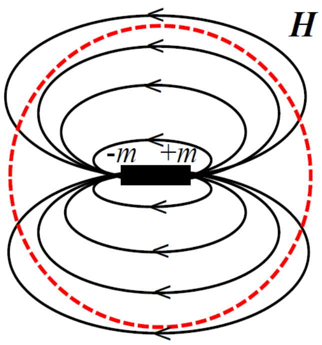



**Fig 3.14** The magnetic field lines from a bar magnet

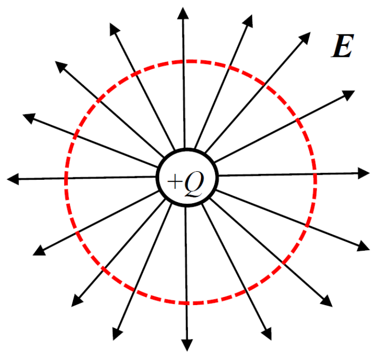



**Fig 3.15** The electric field lines from a charge

- (continuation) It is known that an electric field is generated by a changing magnetic field through electromagnetic induction. The corresponding equation considering the orientation of the fields $\boldsymbol{H}$ and $\boldsymbol{E}$ is

$$\mathrm{rot}\boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} \qquad \text{(C)}$$

where $\mu_0$ is again the magnetic permeability in vacuum. On the other hand, in the case of a changing electric field inside a conductor, an electric current is induced. Even when there is no conductor around, a displacement current is generated which in turn creates a magnetic field. The corresponding equation is

$$\mathrm{rot}\boldsymbol{H} = \boldsymbol{I} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \qquad \text{(D)}$$

The four equations (A) $\sim$ (D) together are known as the Maxwell's equations, the most fundamental equations in electromagnetism.

# Experiment 4. Thermionic Emission Experiment

## §1 Experimental Objective

Electrons are emitted from the surface of metals held at high temperatures in a process known as thermionic emission. The primary objective of this experiment is to investigate the physics of thermionic emission using a dynode vacuum tube.

## §2 Overview

You will empirically verify the equations describing the temperature-limited and space-charge-limited operation states of a diode vacuum tube. At the same time you will obtain the work function of the diode's tungsten filament. In addition, you will gain an understanding of how non-contact temperature measurements of high-temperature objects are made using Planck's blackbody radiation principle.

## §3 Experimental Principle

Conduction electrons inside a metal crystal move freely within it, unaffected by the positive ions which form the crystal lattice. For this reason, these electrons are called **free electrons**. The electrons within a metal have an energy distribution like the one shown in Figure 4.1. The vertical axis of the figure shows the electron energy, the right side of the horizontal axis denotes the distance from the metal's surface, and the left side of the horizontal axis

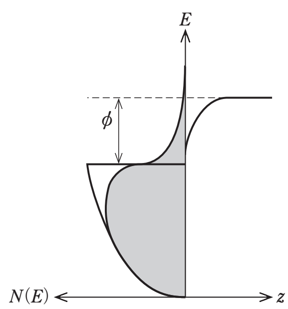



**Fig 4.1** Energy distribution of electrons inside a metal.

shows the number of electrons. Two curves on the left side of the figure show the energy distribution of electrons within a metal held at absolute zero (black curve) and at a finite temperature (gray shaded). As shown in the figure, since there exists a fixed energy difference ($\phi$) between the inside and outside of the metal's surface, a corresponding amount of work is required for a free electron to escape from the surface. This energy difference is known as the **work function** and it differs for different types of metals. When a metal is heated to high temperature, electrons with more kinetic energy than the work function will be emitted from the surface. Electrons emitted in this way are called **thermions** and this process is called **thermionic emission**.

Assuming the electrons in a metal obey a Fermi-Dirac distribution the thermionic current density, $j_s$, which represents the number of electrons emitted from the surface per unit area per unit time, can be written as

$$j_s = AT^2 \cdot \exp\left(-\frac{\phi}{k_B T}\right), \tag{4.1}$$

where $T$ is the absolute temperature and $A = 4\pi mek_B^2/h^3 = 1.2 \times 10^6 \text{A m}^{-2}\text{K}^{-2}$. Here $m$ and $e$ are the electron mass and charge, respectively, $k_B$ is the Boltzmann constant, and $h$ is Planck's constant. This equation is known as the Richardson-Dushman equation.

A vacuum tube is an evacuated glass tube (pressure $\leq 10^{-6}$ mmHg) in which thermionic emission is used to pass a current between two electrodes. The most basic vacuum tubes consist of a thermion-emitting negative electrode, known as the cathode or filament, and positive electrode, known as the anode or plate, which accelerates and collects the thermions. This type of vacuum tube is called a diode vacuum tube and will be used in this experiment.

Diode vacuum tubes have two different thermionic emission states (also called operation regions). First, note that there is an electric field between the anode and cathode due to their different charges. When the number of thermions is small enough that the electric field near the surface of the cathode is unaffected by their charge density all of the emitted electrons (Figure 4.2 a) will flow into the anode. In this case the current in the vacuum tube does not depend on the anode voltage and is only a function of the cathode temperature, as in Equation (4.1). As a result, the relationship between the current and electric potential of the anode is represented by the thin horizontal lines in Figure 4.3. This state is called **temperature-limited** and the corresponding current is known as the **temperature-limited current**.

However, when the density of emitted electrons is high, the cathode becomes surrounded by a cloud of thermions (this is called **space charge**) which are continuously being emitted from and reabsorbed into the cathode. As a result, the electric field at the cathode surface

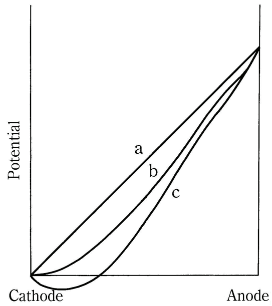



**Fig 4.2** Electric potential inside a diode vacuum tube

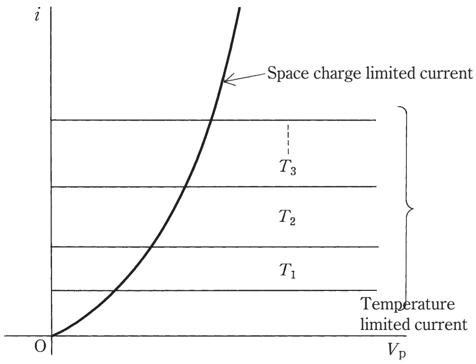



**Fig 4.3** Thermionic current as a function of anode voltage.

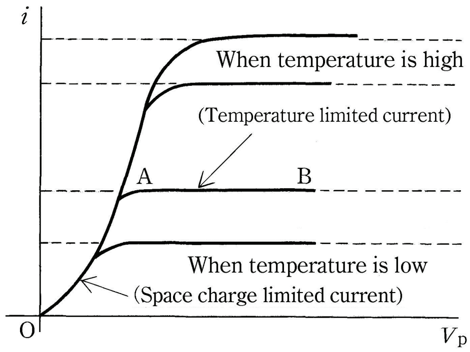



**Fig 4.4** Current response of a diode vacuum tube

becomes weak and in some cases negative (Figure 4.2 b, c). Only those electrons that are sufficiently far from the cathode are drawn to the anode. In this case the electric field is determined by the amount of space charge. Even if an attempt is made to increase the number of thermions by raising the cathode temperature, the space charge will repel additional thermions and above a certain threshold no additional current will flow. As the anode's electric potential is increased, more electrons will flow from the space charge region to the anode. For this reason, the electric current depends on the anode voltage, $V_p$, but not on the cathode temperature. This operation state is called **space charge limited** and the resulting current is called the **space charge current**.

When the anode and cathode are concentric cylinders the space-charge-limited current per unit area, $j$, is given by the **Langmuir equation**, which is also known as the **3/2 power**

**law**:

$$j = 2.33 \times 10^{-6} \cdot \frac{V_p^{3/2}}{ra^2\beta^2} (\text{A m}^{-2}). \tag{4.2}$$

Here $r_a$ is the anode radius, and $\beta$ is a function of the ratio of the anode and cathode radii, $r_a/r_c$. If $r_a/r_c > 10$ then $\beta \sim 1$ [1].

The relationship between the current and voltage in Equation (4.2) is shown by the thick line in Figure 4.3. The current-voltage relationship of a diode vacuum tube will change depending upon the tube's operation state as shown in Figure 4.4. For a given temperature the current will be space charge limited and obey the 3/2 power law (Figure 4.4 O-A) and as the anode voltage increases the tube will transition into the temperature-limited state, after which the current will saturate (Figure 4.4 A-B) at a value determined by the cathode temperature (temperature-limited current).

## §4 Equipment

(a) **Thermionic emission tube (diode vacuum tube)** The vacuum tube is composed of a tungsten cathode (0.15-0.20 mm $\phi$) K, which extends along the axis of the cylindrical anode (plate) P. A hole has been cut in the center of the anode so that the cathode temperature can be measured using an optical pyrometer. See Figure 4.6.

(b) **Optical pyrometer** An optical pyrometer is a device which determines the temperature of an object by measuring the intensity of thermal radiation at a fixed frequency (in the visible part of the spectrum) emitted by the object (Figure 4.5). It operates based on the Stefan Boltzmann blackbody radiation law, which states that the energy of thermal radiation emitted from a blackbody is proportional to its temperature raised to the fourth power.

## §5 Method

(1) Arrange and wire the thermionic emission tube, power supply, and meters as shown in Figure 4.7. Connect the meters so that, in general, inflowing current and higher voltages go to the (+) terminal, and so that outflowing currents and lower voltages leave from the (−) terminal. The milliammeter, the ammeter, and the voltmeter must be oriented so that their scales are upright.

The following are instructions for using the constant voltage power supply ($P_1$) and the constant current power supply ($P_2$). A constant voltage power supply is a power supply that provides a constant voltage even if the current through its load changes.

[1] When $\beta = 1$ Equation (4.2) reduces to that for parallel plate electrodes.

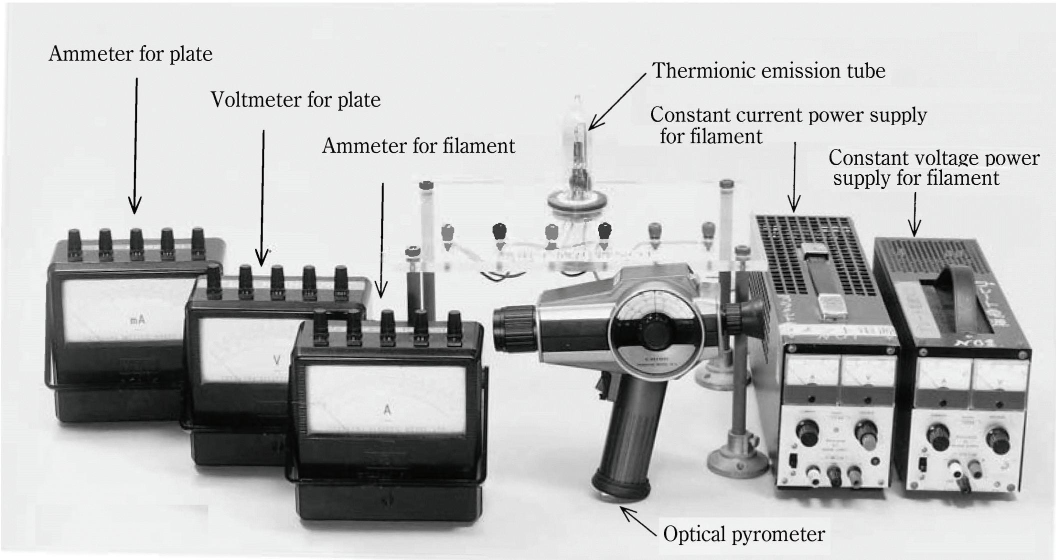



**Fig 4.5** Equipment for the thermionic emission experiment.

On the other hand, a constant current power supply provides a constant current even if the voltage across its load changes.

$P_1$ Turn the current adjust dial to its maximal value (rotate fully clockwise), then turn the voltage adjust dial to its minimum value (rotate fully counterclockwise) and then slowly increase the voltage adjust to the desired voltage.

$P_2$ Set the voltage adjust dial to its minimum and then increase it until the desired current is obtained.

In this experiment the ammeter will be used at a full scale of 3 A or 10 A and the

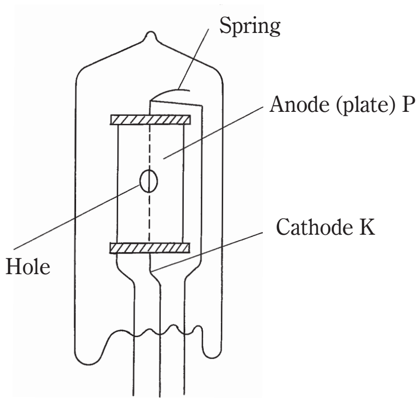



**Fig 4.6** Diode vacuum tube.

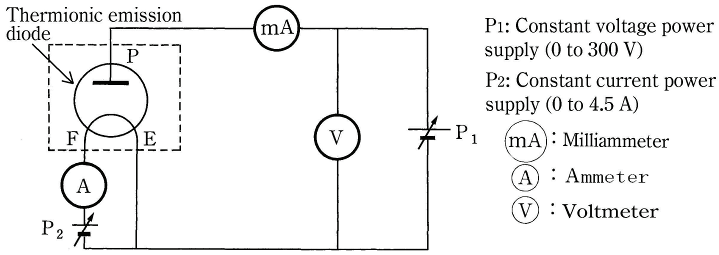



**Fig 4.7** Wiring diagram

voltmeter at a full scale of 300 V. However, since the anode current changes with the applied voltage and the temperature of the cathode, the milliammeter's (−) terminal connection (full scale) should be changed as needed.

Place the meters upright on the desk, set the (−) terminal of the milliammeter, and set the constant voltage power supply voltage to zero.

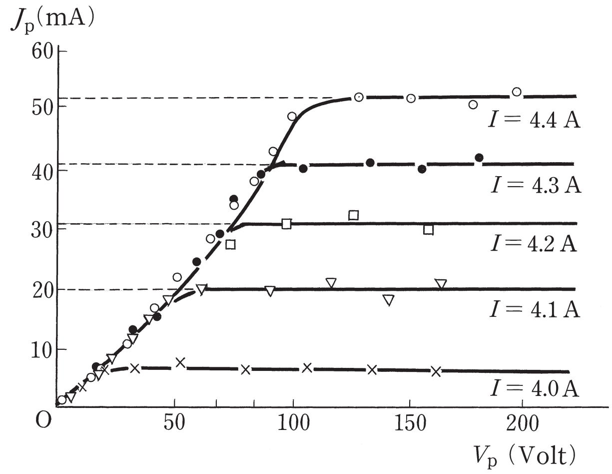



**Fig 4.8** Anode current as a function of voltage.

(2) The primary goal of this experiment is to obtain data like those shown in Figures 4.9 and 4.8. Though either data set may be obtained first, it is important that the cathode current $I$ be in the same range for both measurements.

Let's take a brief look at the characteristics of Figure 4.8. Set the cathode current, $I$, to any reasonable value so that the filament of the diode vacuum tube (the cathode in Figure 4.6) is incandescent (glowing). Typically the current will be more than 4.0 A. At this setting slowly raise the voltage on the constant voltage power supply while paying attention to the anode current measured by the milliammeter, $J_p$. Visually verify that at low voltages the current increases rapidly but later saturates, much like in Figure 4.8. Find and fix the voltage at which the current is nearly saturated and record the approximate value of the anode current $J_p$. Divide this current into five equal parts, $J_{p,i}$, and reduce the initial cathode current without adjusting the anode voltage to find the five cathode current values, $I_i$, that give anode currents closest to these five values. Record each of the cathode current values, $I_i$. They will be used to perform the measurements in steps (3) and (4) below.

(3) For each cathode current value $I_i$ prepared in step (2), change the anode voltage $V_p$ from 0 V to about 200 V in small steps and record the value of both the voltage and the measured anode current $J_p$ at each step. At low voltages the changes in $J_p$ will be large and small steps are needed, but at higher voltages it will stop changing rapidly and larger steps can be used. Thermions have an initial velocity which means that even when $V_p = 0$ there should be a small anode current (initial velocity current). Determine whether or not this current can actually be detected.

(4) In addition to the measurements in step (3), use the optical pyrometer to measure the filament temperature $T$ keeping $V_p = 0$. Measure the temperature at 10 different cathode currents over a range that includes the currents $I_i$ used in step (3). Though the optical pyrometer is a useful tool, it is nonetheless easy to introduce measurement errors. Be sure that the pryometer is properly focused on the hottest part of the filament. Firthu, the overall precision can be increased by making repeated temperature measurements at each current value. Plot the repeated measurements as a bar, like those shown in Figure 4.9, and draw a smooth curve through those bars to create the cathode current versus temperature ( $I$ vs. $t$ ) calibration curve for your diode vacuum tube.

(5) When all measurements have been completed, return all of the power supply dials to their zero position by fully rotating them counterclockwise and then turn off the power supplies.

(6) Disconnect all wires from the experimental apparatus and return them to their original location.

## §6 Analysis Topics for Report

(1) Determine the absolute temperature of the cathode from the calibration curve in step (4) above and using graph paper draw characteristic curves by plotting $J_p$ and $V_p$ values for each temperature.

(2) In the space-charge-limited regime Equation (4.2) is valid. Taking the standard loga-

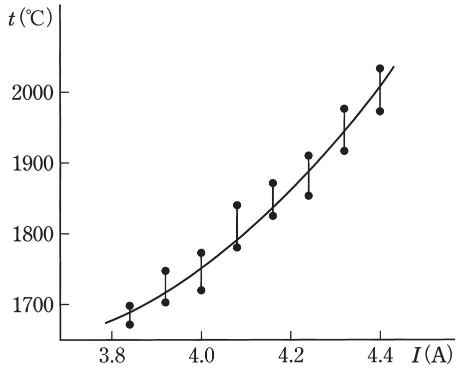



**Fig 4.9** Example cathode temperature versus current calibration curve.

rithm of both sides yields,

$$\log_{10} J_p = \frac{3}{2} \log_{10} V_p + \text{const.} \tag{4.3}$$

Accordingly, the relationship between $\log_{10} J_p$ and $\log_{10} V_p$ is linear with an expected slope of 3/2.

Plot all measured $J_p$ and $V_p$ values on a logarithmic plot, obtain the slope of the line in the space-charge-limited region and verify whether or not the relationship shown in Equation (4.3) holds. (An explanation of logarithmic plots can be found in Chapter IV, subsection 3-c.)

(3) The following relationship can be obtained from Equation (4.1) in the same way :

$$\log_{10} \frac{j_s}{T^2} = \log_{10} A - 0.434 \frac{\phi}{kT}. \tag{4.4}$$

Here $j_s$ is the saturation current density. If $J_s$ denotes the measured saturation current and $S$ denotes the area of the surface from which the thermions are emitted, then $j_s = J_s/S$. Further note that the saturation current value at $V_p = 0$ is determined by extrapolating the trend in the anode current after its increase has become very gradual (dashed lines in Figure 4.8). The radius of the cathode in this experiment is 0.2 mm. If we assume that the emitted electrons those which contribute to the anode current are primarily emitted from a surface along the length of the anode then the effective length of that surface is 2.0 cm.

For each current $I_i$ above, organize the corresponding values for $T$, $1/T$, and $j_s/T^2$ in a table taking into consideration the variation in the measured temperatures (in absolute units). In addition, for each data set take $j_s/T^2$ and $1/T$ to be the vertical and horizontal axes of a semi-log plot and plot your measured values. (For information about semi-log graphs see Chapter IV subsection 3-b.)

Assuming the points lie on a straight line described by Equation (4.4), use its slope to determine the work function $\phi$ of tungsten in units of electron-volts (eV). Since there are variations in both the line and its slope there should also be variation in the measured value of $\phi$. Evaluate the work function including those variations. Next, substitute your measured $\phi$ and any value from your plotted experimental line into Equation (4.4) and compute the value of $A$.

For reference, Tungsten's work function is 4.55 eV and the theoretical value of $A$ is $1.2 \times 10^6$A $\mathrm{m}^{-2}\mathrm{K}^{-2}$. Using Equation (4.1) consider possible reasons for errors in your measured value of $A$.

(Reference Material)

Even in the temperature-limited regime, $J_p$ will increase slightly with increasing $V_p$. Though there are many reasons for this, the mechanism that can quantitatively account for nearly all of the phenomenon is thought to be the "Shottkey effect." The Shottkey effect is a phenomenon in which the apparent work function of a metal surface in the presence of a strong electric field is reduced due to tunneling. For the thin-filament cathode used in this experiment, the electric field strength at the cathode's surface is inversely proportional to filament radius. This electric field increases the current $J_p$ by a few percent. The Shottkey effect is important an important part of the operating principle of tunnelling microscopes.

(Supplement)

Since the optical pyrometer has been calibrated for blackbody radiation, it will generally report temperatures lower than the true temperature and must be corrected when the object cannot be treated as a perfect blackbody. Strictly speaking the cathode used in this experiment cannot be considered a blackbody and its actual emission state must be considered to determine the correct temperature. However, in this experiment the error induced by not making this correction is small.

# Experiment 5. Experiments using Lasers

## §1 Experimental Objective

We will try to understand the properties of light through experiments using single-slit and polarization filters.

## §2 Overview

Lasers are source of coherent and (almost) monochromatic light. “Coherent” waves are interfering waves whose waveforms are in-phase for a prolonged period of time and over a long range in space. Laser light is highly collimated and unidirectional, so it is able to propagate a long distance without much dispersion. High intensities are realized by focusing light into a small cross section area. Furthermore, it can be made polarized. We will perform two experiments to explore these unique properties of laser light.

## §3 Experiment 1 - Light diffraction through a single-slit

In this experiment, we will diffract some laser light using a single-slit. We will measure the intensity distribution of the diffracted light and compare that with the theoretical prediction.

(a) **Theory** As an example of diffraction, imagine that a plane wave is directed

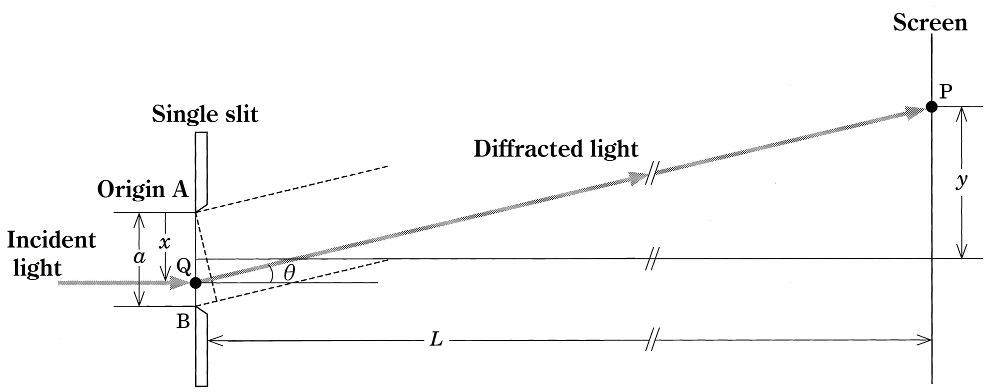



**Fig 5.1** Interference of light by a single-slit

toward a single-slit of width $a$, and the intensity of the diffracted wave is measured behind the slit by placing a projection screen at a sufficiently long distance. Here, we assume that the slit is infinitely long. Figure 5.1 shows the vertical cross section of the slit and screen. If the incident light from the laser is a plane wave, the phase of the wave must be the same everywhere on AB. As in Figure 5.1, consider the diffracted wave on a point P on the screen which makes an angle $\theta$ with the incident direction. This diffracted wave is the linear combination of all the waves sent out from across the slit AB. The intensity $I_{\mathrm{P}}$ of the composite wave at this point P is thus

$$I_{\mathrm{P}}(\theta) = |\phi_{\mathrm{P}}|^2 = \frac{C^2\lambda^2}{\pi^2 \sin^2\theta}\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right) \tag{5.1}$$

Here $\phi_{\mathrm{P}}$ is the combined amplitude of all the wave from AB reaching point P (please refer to the Reference section "Calculation of the amplitude of composite wave from a single-slit" for the derivation of Equation 5.1). Now, we can define $\alpha \equiv \dfrac{\pi a \sin\theta}{\lambda}$ so that

$$I_{\mathrm{P}}(\theta) = C^2 a^2 \frac{\sin^2\alpha}{\alpha^2} \tag{5.2}$$

From this, we can see that when $\alpha = m\pi$, i.e., when $\theta$ satisfies

$$a\sin\theta = m\lambda \qquad (m \text{ is any integer other than } 0) \tag{5.3}$$

, the intensity $I_{\mathrm{P}}$ is at its minima 0. Let the distance between the diffraction slit and the screen be $L$, and $y$ be the distance between point P and the intersection point of the incident direction and the screen (see Figure 5.1), then

$$\tan\theta = \frac{y - a/2 + x}{L} \tag{5.4}$$

If $y \ll L$, $\sin\theta \simeq \tan\theta \simeq \dfrac{y}{L}$, $\alpha \simeq \dfrac{\pi a y}{\lambda L}$ , the intensity $I_{\mathrm{P}}$ as a function of $y$ becomes

$$I_{\mathrm{P}}(y) = C' \left\{ \frac{\sin\left(\frac{\pi a}{\lambda L} y\right)}{\frac{\pi a}{\lambda L} y} \right\}^2 \tag{5.5}$$

Here, we have defined $C' \equiv C^2 a^2$. Furthermore, the criterion for minimum intensity (Equaion 5.3) becomes

$$y = m\frac{\lambda L}{a} \tag{5.6}$$

Similarly, from Equation 5.2, the intensity reaches its local maxima when $\alpha\cos\alpha - \sin\alpha = 0$, i.e., when $\alpha = \tan\alpha$ is satisfied. From numerical calculations, one can find the locations on the screen where $I_{\mathrm{P}}$ is at its local maxima. For convenience, one can renormalize the intensity of the diffracted light in unit of $C'$ (i.e., $I_{\mathrm{P}}/C'$), so that the maxima and minima intensities can be tabulated like in Table 5.1. Here, $n$ and $m$ are the sequential order of bright fringes and dark fringes respectively, counting from the center of the screen where $n = m = 0$.

Figure 5.2 shows the expected intensity distribution of the diffracted laser light on a screen

**Table 5.1** Renormalized values of maxima and minima of $I_{\mathrm{P}}/C'$

| Bright fringe order $n$ | 0 | | 1 | | 2 | | 3 | | 4 |
|---|---|---|---|---|---|---|---|---|---|
| Dark fringe order $m$ | | 1 | | 2 | | 3 | | 4 | |
| $\alpha(\equiv \pi a y/\lambda L)$ | 0 | $\pi$ | $1.43\pi$ | $2\pi$ | $2.46\pi$ | $3\pi$ | $3.47\pi$ | $4\pi$ | $4.48\pi$ |
| $I_{\mathrm{P}}/C'$ | 1.000 | 0 | 0.047 | 0 | 0.017 | 0 | 0.008 | 0 | 0.005 |

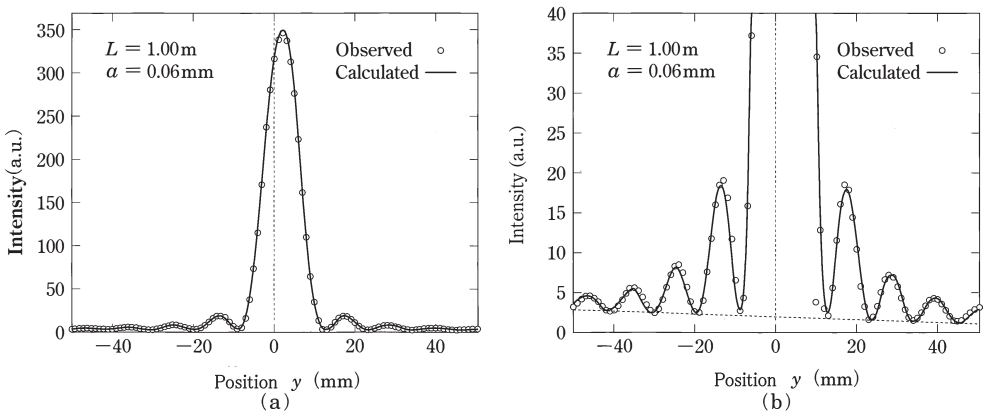



**Fig 5.2** Intensity distribution of diffracted light from a single-slit ($L = 1.00$ m, $a = 0.06$ mm). Panel (b) is a zoom-up version of (a) to better visualize the local maxima.

in the case of a single-slit. The ∘ markers are examples of data points obtained from this experiment, and the solid line is sum of the theoretical prediction (Equation 5.5) and the background intensity (dashed line in Figure 5.2 (b)). We can see that the above theory (Equation 5.5) is highly compatible with experimental results here.

(b) **Experimental devices** In this experiment, we will direct a laser light generated by a laser oscillator onto a single-slit, and measure the intensity distribution of the diffracted light at a distance $L$ from the slit using a photodiode. The intensity can be estimated by measuring the output voltage in a digital multimeter as a result of the generated current in the photodiode. The devices we will employ include the following:

(1) Laser oscillator: a He-Ne laser with a power of 2 mW and wavelength $\lambda = 632.8$ nm. The package consists of the oscillator itself and its power supply.

(2) Optical system for laser light interference: as shown in Figure 5.4, a laser oscillator, a diffraction slit (single-slit), and a light reception slit with photodiode are mounted on an optical bench. Their carriers can be moved around and tightened in desired positions. The light reception unit (e.g., the photodiode) is placed on a stage movable along an axis parallel to the laser incident direction, and can be displaced along an axis on the screen ($y$-axis) in the range of -60 mm to + 60 mm.

(3) Light detector: we will employ a Si photodiode (S2386-8K). A photodiode is a device which generates a photoelectric current proportional to the incident light intensity. This photoelectric current can be measured by passing it through a serially connected loading resistance. You will measure the resulted voltage and convert it back to the light intensity. As long as the voltage is lower than roughly 300 mV, it is approximately proportional to the intensity of the diffracted laser light.

(4) Digital multimeter (HP34401A): we will use it to measure the voltage across a loading resistance in this experiment. See Figure 5.5 for its interface. By connecting the loading resistance to the LO and HI ports, you will be able to measure the light intensity as a voltage value.

(c) **Measurements**

[**Caution**]

(1) Never, ever, point the laser into your naked eyes!

(2) Turn the adjustment screw of the single-slit gently! In particular, you must not fasten the screw beyond the 0 mark after the slit is already fully closed.

(c-1) Preparation

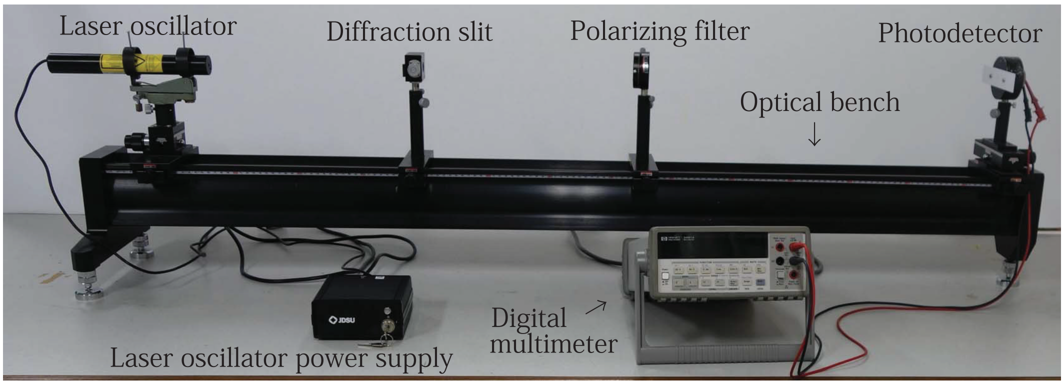



**Fig 5.3** Device setup for the laser experiment

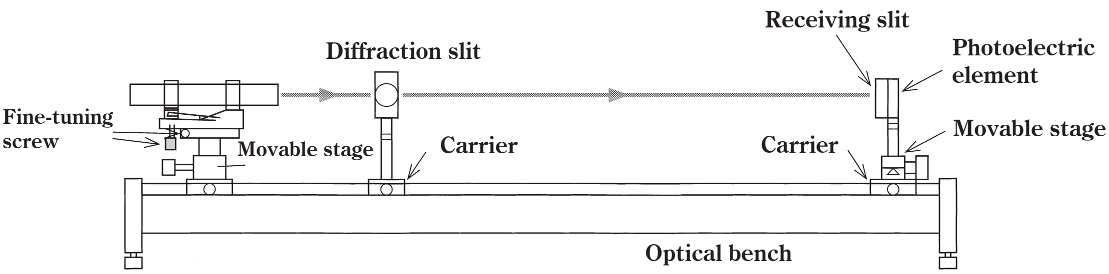



**Fig 5.4** Optical system for laser interference

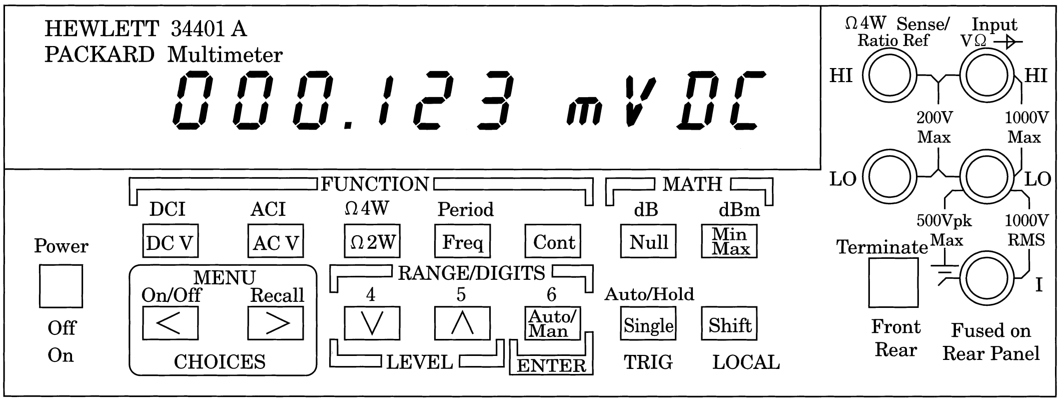



**Fig 5.5** The front panel of the digital multimeter (DMM)

Set up the diffraction slit and detector (photodiode) so that they are separated by a distance of 1.00 m. Align the movable stage of the detector to the origin. Make sure the diffraction slit and the reception surface of the photodiode are placed at the center of the carrier. Open up the single-slit widely, turn the micro-adjustment screw of the laser holder and adjust the movable stage until the laser light points at the center of the detector. Then, close up the single-slit to make a narrow slit. Further adjust the laser holder alignment until the incident laser light is uniform across the slit opening, and a symmetric diffraction pattern is observed on the screen. Finally, tune the slit opening until the separation between the bright and dark fringes on the pattern is roughly 10 $\sim$ 20 mm.

(c-2) Measurement of diffraction profile

To determine the actual zero-point of the slit width, narrow up the slit until the diffraction light pattern just disappear. Record this slit width on the markings as $a_0$. Pay attention not to over-tighten the adjustment screw. Stop turning the screw immediately once the pattern vanishes. Then, open the slit to a width of about 0.1 mm (eye measurement is fine in this stage). Connect the output connectors of the photodiode to the digital multimeter. Adjust the movable stage until the voltage reading (light intensity) is maximum. Confirm that this maximum reading is roughly 300 mV. If it is higher than 300 mV, further decrease the slit width until the reading drops to about 300 mV. Under this condition, record the slit width from the markings as $a_1'$. Take the difference $a_1 = a_1' - a_0$ to be the effective slit width. Then, in steps of 1mm, displace the movable stage from -50 mm to +50 mm, and record the voltages on the multimeter for each position to measure the laser intensity distribution.

(Note) On Vernier scale: The smallest marking on the main scale is 1 mm. The secondary (Vernier) scale is split into 50 equal parts, so the smallest scale is $\frac{1}{50}$ mm. Read § II-1(c) to learn how to read a Vernier scale.

**Once again, never let a laser light enter your eyes! Be especially cautious when you read the scale on the movable stage.**

(d) **Data analysis and Discussion**

(d-1) Determining the slit width

Plot the data you obtained in (c-2) with the distance $y$ on the horizontal axis and voltage on the vertical axis, similar to Figure 5.2. In resemblance with panel (a) and (b) in Figure 5.2, first make a graph for the whole data set, and then make another graph to zoom into the secondary bright fringes.

Following that, read off from the graph above the position $y_m$ of the minimal intensities, and plot the relation between position $y_m$ and diffraction order $m$ on a graph paper. Confirm that $y_m$ is linearly related to $m$ as predicted by Equation 5.6 Then, calculate the slope $(\lambda L/a)$ of the relation from the graph, and from that estimate the slit width $a$. Compare that with $a_1$ that you measured in (c-2).

While theory predicts that the minimum values should be exactly zero, in reality you will get non-zero readings in your experiment. Various things like dark current in the photodiode and astray light contribute to the background $I_{\mathrm{g}}$ of the diffracted laser light intensity. Since the position-dependence of $I_{\mathrm{g}}$ highly depends on the nature of the main contributor to the background, in general it is non-trivial how to write it down in a functional form. However, we can still approximate it as a simple function of $y$. Namely, draw a straight line through the minimum points of the intensity distribution graph and find its formula. You will need to subtract this background from your data as you discuss the intensity of diffracted light below.

(d-2) Estimation of the maximum values

From your graph, find the intensities of the maxima for each order $n$ (background subtracted) and their corresponding positions. Tabulate these values. Next, renormalize these intensity values and estimate the positions relative to that of the zeroth order maximum, and make another table. Compare this table with Table 5.1.

(d-3) Profile comparison

Into Equation 5.5, substitute the slit width $a$ from (d-1), the distance $L$ between the slit and the screen, the wavelength $\lambda$ of your laser light source, and the zeroth order peak intensity $C'$. Find the formula for the intensity of the diffracted light as a function of $y$. Do not forget to add to this formula the background function you found in (d-1). Plot this formula on your graph and compare it with your experimental data. If the scale origin

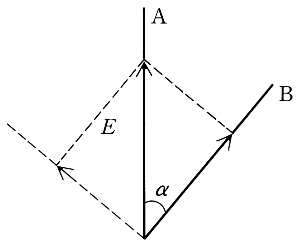



**Fig 5.6** Concept diagram for polarization

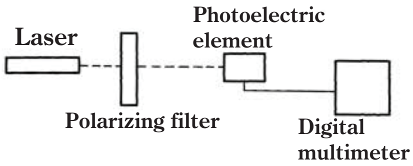



**Fig 5.7** Concept diagram for the measurement

($y = 0$) of the movable stage is not exactly aligned with the the position of the zeroth order peak, you may need to readjust your curve to make them match.

## §4 Experiment 2 - Polarization

**Confirmation of linear polarization using a polarization filter**

(a) **Introduction**

In the atmosphere, light is a transverse wave such that the electric field ($\vec{E}$) and magnetic field ($\vec{B}$) vectors are on planes perpendicular to the propagation direction. Since $\vec{B}$ is orthogonal to $\vec{E}$, we focus our discussion on $\vec{E}$ here. Light coming out of an ordinary light source is basically a combination of waves with various orientations of $\vec{E}$. On the other hand, laser lights, such as the one we are using in this experiment, have their $\vec{E}$ oriented in a fixed direction. These kind of light is regarded as having a linear polarization. To confirm this assertion, you will measure the laser light intensity through a polarization filter. A polarization filter, as its name implies, filters out waves of which $\vec{E}$ is misaligned with a certain fixed orientation.

As in Figure 5.6, let A be the direction of $\vec{E}$ of the laser light, and B be the direction of $\vec{E}$ that can pass through the polarization filter. The component of $\vec{E}$ of the laser light along direction B is $E\cos\alpha$. Since the light intensity is proportional to $|\vec{E}|^2$, the intensity of light passing through the filter is proportional to $\cos^2\alpha$.

(b) **Experimental devices and method** First, remove the diffraction slit and its carrier from the bench. Mount the polarization filter and its carrier onto the bench in replacement (see Figure 5.7 for illustration). Rotate the filter until the voltage reading on the multimeter reaches its minimum value. Record the rotation angle of the filter as $\theta_0$, and the minimum voltage as $V_0$. Then, increase the angle in steps of 5 degrees, and record the corresponding voltage readings as $V$. The minimum $V_0$ can be understood as contributions from ambient light other than the laser and dark current of the photodiode.

(c) **Data analysis and Discussion** Since $\sin(\theta - \theta_0) = \cos\alpha$, one can expect that $V - V_0$ is proportional to $\sin^2(\theta - \theta_0)$. On a graph paper, plot your measured values $V - V_0$ (vertical axis) against $\sin^2(\theta - \theta_0)$ (horizontal axis). If there is any data point which deviate from a linear relation, discuss the possible reason(s).

## §5 ⟨Reference⟩ Calculation of the amplitude of composite wave from a single-slit

Here we elaborate on the derivation of Equation 5.1. Let $a$ be the slit width, and P be a point on a screen sufficiently distant from the slit. We can understand the light projected on P as the combination of light from all points in the slit diffracted by different angles $\theta$ (see Equation 5.4). In Figure 5.1, compared with wave emitted from point A, wave emitted by the infinitesimal region $x \sim x + \mathrm{d}x$ would travel along a path $x \sin\theta$ longer, which is $2\pi \frac{x \sin\theta}{\lambda}$ (radian) lagged behind in phase if $\lambda$ is the wavelength of the laser light. As a result, if $\xi$ is the phase difference between point A and P, the amplitude of the composite wave contributed by all points along AB can be written as

$$\phi_{\mathrm{P}} = C \int_0^a \cos\left(\xi - 2\pi \frac{x \sin\theta}{\lambda}\right) \mathrm{d}x \qquad (5.7)$$

, where $C$ is a constant proportional to the amplitude of the incident wave from the laser. To carry out the calculation here, it is convenient to use the Euler formula

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta \qquad (5.8)$$

to express it in exponentials. $\mathrm{i} = \sqrt{-1}$ is the complex unit, and $\mathrm{e}^z$ is an exponential function. In this way, the integral in Equation 5.7 can be rewritten as

$$\cos\left(\xi - 2\pi \frac{x \sin\theta}{\lambda}\right) = \Re\left[\mathrm{e}^{\mathrm{i}\left(\xi - 2\pi \frac{x \sin\theta}{\lambda}\right)}\right] = \Re\left[\mathrm{e}^{\mathrm{i}\xi}\, \mathrm{e}^{-2\pi\mathrm{i} \frac{x \sin\theta}{\lambda}}\right] \qquad (5.9)$$

, where $\Re[z]$ is the real part of a complex number $z$. The amplitude $\phi_{\mathrm{P}}$ of the composite wave at point P in Equation 5.7 is guaranteed to be a real number, so that we can omit the $\Re[\ ]$ notation, and it becomes

$$\phi_{\mathrm{P}} = C\mathrm{e}^{\mathrm{i}\xi} \int_0^a \mathrm{e}^{-2\pi\mathrm{i} \frac{x \sin\theta}{\lambda}} \mathrm{d}x = -\frac{C\lambda \mathrm{e}^{\mathrm{i}\xi}}{2\pi\mathrm{i}\sin\theta} \left(\mathrm{e}^{-2\pi\mathrm{i} \frac{a \sin\theta}{\lambda}} - 1\right) \qquad (5.10)$$

Since the intensity of light is proportional to the square of its wave amplitude, the intensity of the diffracted light at point P is

$$I_{\mathrm{P}}(\theta) = |\phi_{\mathrm{P}}|^2 = \frac{C^2\lambda^2}{\pi^2 \sin^2\theta} \sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right) \qquad (5.11)$$

, which is exactly what we have in Equation 5.1.

# Experiment 6. Measurement of Wavelength of Light using a Diffraction Grating

## §1 Motivation

In this experiment, we will try to understand the basic principles of a diffraction grating monochromator. In doing so, we will perform diffraction angle measurements to estimate the wavelength of a spectral line (D-line) emitted by Na atoms.

## §2 Introduction

Diffraction gratings are widely used along with prisms in spectrometers for optical spectroscopy. In principle, diffraction gratings are devices that separate light into its components of different wavelengths by passing it through equally spaced narrow slits from which the diffracted lights interfere with each other.

## §3 Principles

As shown in Figure 6.1, consider an incident light of wavelength $\lambda$ directed onto a diffraction grating G perpendicular to its plane. Let $d$ be the spacing between the slits (i.e., gratings), and *theta* be the diffraction angle relative to the normal of the grating plane. When we observe at $\theta = 0$, it is easy to see that the transmitted lights from all the slits are in-phase and hence interfere constructively with each other. When $\theta \neq 0$, the transmitted

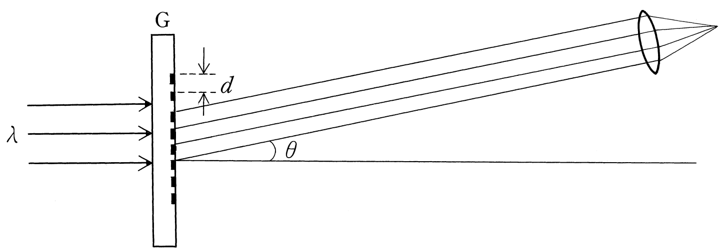



**Fig 6.1** The working principle of a diffraction grating

lights from adjacent slits have a difference of $d\sin\theta$ in path length, so that when $\theta$ satisfies

$$d\sin\theta = 2m\left(\frac{\lambda}{2}\right)\ (m = 1, 2, 3, ...), \tag{6.1}$$

the transmitted lights are again in-phase and reinforce each other. As a result, we can expect a series of bright fringes with order $m = 1, 2, 3, ...$ generated on both sides of the brightest zeroth order ($m = 0$) line along the direction of the incident light. The diffraction angle $\theta_\mathrm{m}$ of the $m$-th order fringe can be calculated from Equation 6.1 to be

$$\lambda = \frac{d\sin\theta_\mathrm{m}}{m} = \frac{\sin\theta_\mathrm{m}}{mn} \tag{6.2}$$

Here, $n = 1/d$ is the number of slits within a unit length along the grating plane. If $n$ (or equivalently $d$) is known, we can know the wavelength $\lambda$ of the incident light by making measurements of the diffraction angles $\theta_m$ of the bright fringes.

Now consider two incident lights (spectral lines) of slightly different wavelengths $\lambda$ and $\lambda + d\lambda$. Then as illustrated in Figure 6.2, there is a limitation for any particular diffraction grating on its ability to resolve these two lines and tell their wavelengths apart. When such a limitation is reached, we can define $\lambda/d\lambda$ as the resolving power of the grating.

This "resolving power" depends on the observed spectral line width $W$ resulted from the superposition of diffracted lights from all the slits on the grating. This width $W$ can be made smaller by packing more slits into the grating. From a theoretical treatment, we can deduce that the resolving power can be written as

$$\lambda/d\lambda = mN \tag{6.3}$$

where $N$ is the total number of slits through which the incident light passes through (see, e.g., Chapter 8 of Principles of Optics, M. Born & E. Wolf). From this we can see that high precision spectroscopic measurements can be achieved by using diffraction gratings of larger total slit number $N$.

Instead of using slits as introduced above, diffraction gratings most often seen today are made of planar glasses with parallel lines engraved on their surfaces. These engraved lines, which introduce periodic unevenness on the glass, play exactly the same role as the parallel slits with very similar underlying principles. Other types of gratings such as reflection type using glass mirrors and non-planar type with concave or stepped shapes also exist.

The Na D-lines which are yellowish in color are the strongest lines in the emission spectrum of Na atoms. They consist of two lines, namely $D_1$ (589.592nm) and $D_2$ (588.995nm). The electronic structure of a Na atom is $1s^2$, $2s^2$, $2p^6$, 3s. The bound 3s electrons can be excited to the 3p orbital of higher energy by being collided with free electrons from a hot cathode. When they decay spontaneously back to their original 3s state, photons of specific energies

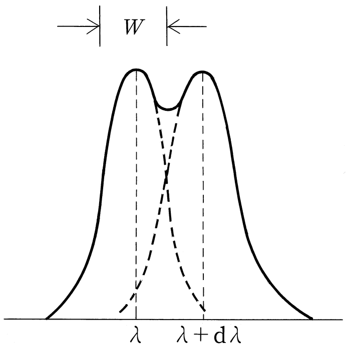



**Fig 6.2** Overlapping D-lines on the verge of being resolvable

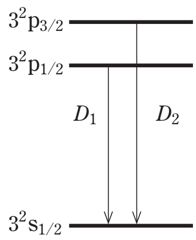



**Fig 6.3** The electronic energy levels of a Na atom

are emitted, and these photons are the origin of the D-lines. As shown in Figure 6.3, since the 3p energy level is further split into two levels ($^2P_{3/2}$, $^2P_{1/2}$) due to a relativistic effect known as spin-orbit coupling, there are two D-lines of slightly different energies.

## §4 Experimental Devices

A layout diagram for the experimental setup is shown in Figure 6.4, and the name for each experimental component in Figure 6.5. A Na lamp is placed in front of the collimator from which parallel light rays emerge and are projected perpendicularly onto a diffraction grating. The diffracted light pattern is then observed through a telemeter (see Figure 6.6). The grating used in this experiment (Figure 6.7) has 2000 parallel engraved lines per 1 cm. It is an extremely delicate and pricey place of instrument, so do not touch the grating surface with your bare hands.

## §5 Measurements

### 1. Tuning the Spectrometer

#### a) General experimental cautions

(1) Make sure that the experiment is performed in a dark room using blackout curtains

(2) The spectrometer has a large number of adjustment screws on it. Make sure you

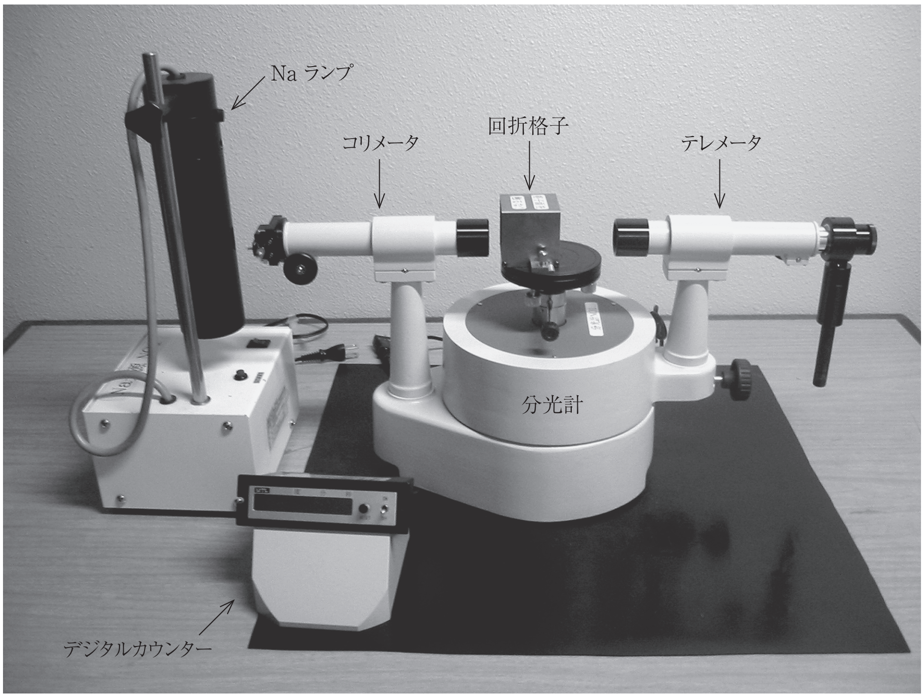



**Fig 6.4** Experimental setup

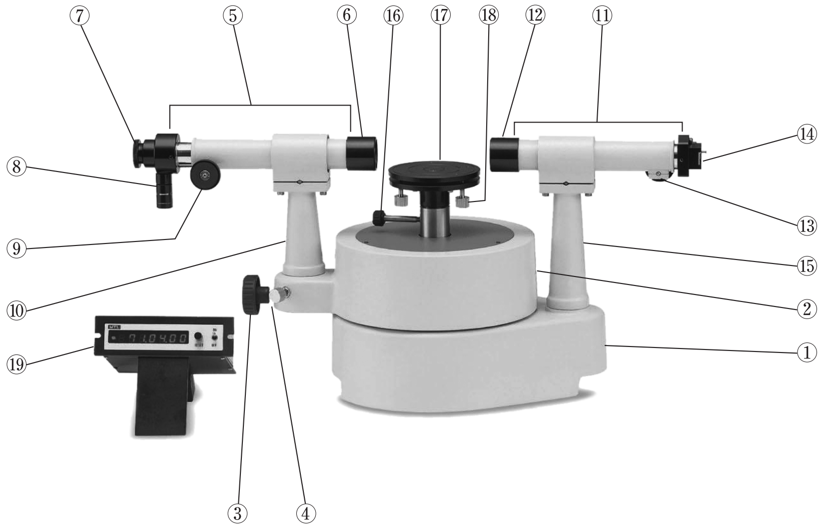
understand the role of each of them, or otherwise you may end up confusing yourself after a long time doing fine adjustments. Use Figure 6.5 effectively to carefully identify the many components on the instrument.

(3) When you orient the telemeter for observations, hold and rotate its support base, not the barrel itself.

(4) The slit on the collimator is a very crucial part of the spectrometer. Do not ever over-tighten its adjustment screw, or do anything careless that would damage the sharp edges of the slit.

(5) To observe the fringes of higher orders, which are relatively faint, it is important that you place the light source directly in front of the collimator, and that the grating lines are as parallel as possible to the slit of the collimator. This ensures that you obtain maximum brightness of the fringes.

**b) Tuning the Spectrometer and Diffraction Grating** As for any experiment, one needs to make appropriate adjustments to the experimental setup to achieve precise measurements. To this end, you will perform a 3-step procedure known as "auto-collimation" widely used in optical experiments:




① ベース
② 回転腕
③ クランプハンドル
④ 微調整ハンドル
⑤ テレメータ(望遠鏡)
⑥ テレメータレンズ
⑦ 接眼レンズ
⑧ オートコリメーション照明
⑨ ピント調整ハンドル
⑩ テレメータ支柱
⑪ コリメータ
⑫ コリメータレンズ
⑬ ピント調整ハンドル(スリット像用)
⑭ スリット
⑮ コリメータ支柱
⑯ ステージ固定ツマミ
⑰ ステージ
⑱ ステージ水平調整ネジ
⑲ デジタルカウンタ

**Fig 6.5** Spectrometer

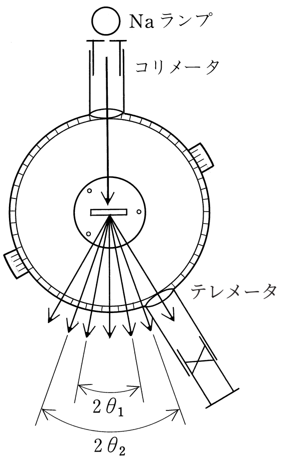



**Fig 6.6** Schematic diagram of the spectrometer (top view)

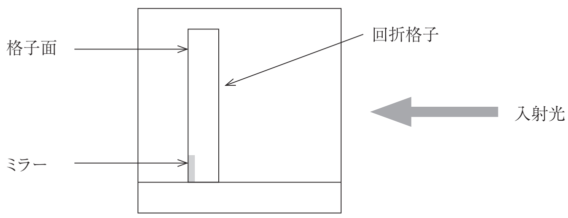



**Fig 6.7** Schematic diagram of the diffraction grating (side view)

(A) Adjust the telemeter (set the focal length to infinity and do eyesight adjustments)
(B) Adjust the optical axis of the telemeter perpendicular to the rotation axis of the diffraction grating
(C) Adjust the collimator

Below describes the details for each step (follow them carefully):

(A) Adjustment of telemeter (infinite focal length & eyesight adjustment)

(1) Line up the telemeter with the collimator as best as you can.
(2) Adjust the stage so that it is as horizontal as possible to your eyes using screw ⑱ (see Figure 6.8). Then, place a spirit level on the stage, and fine tune screw S1 or S2 so that the stage is level along the S1-S2 direction. Put the spirit level away as you are done.
(3) Rotate the stage, so that the mirror surface on the grating (located in the bottom part) is almost perpendicular to the telemeter.
(4) Turn on the auto-collimation light by rotating the LED housing clockwise (from the top). Do not over-turn the housing.
(5) Peer into the eyepiece. Make sure the reflected image of the auto-collimation prism (the dark part near the bottom of your vision) is within and in the upper part of the field of vision (see Figure 1). Then, rotate the eyepiece to focus the cross line (eyesight adjustment). It is recommended that you place a piece of white paper in front of the telemeter to make it easier.
(6) Move the focus adjustment handle ⑨ of the telemeter, so that the reflected image of the cross line (inside the reflected image of the prism, as shown in Figure 1) is focused. When there is a parallax between the cross line and its reflected image, the image should displace relative to the cross line as you move your eyeball left and right. Rotate the focus adjustment handle until there is no parallax at all.

(B) Adjustment to make the telemeter optical axis perpendicular to the stage rotation axis

As in Figure 1, when the left and right gaps e between the cross line and its reflected image are the same, you know that the optical axis of your telemeter is perpendicular to the diffraction grating. Also, as shown in the Figure, make sure the reflected image is inside the cross line. To confirm this, follow the steps below:

(1) Rotate the fine-adjustment handle ④ of the telemeter as you peer into the eyepiece

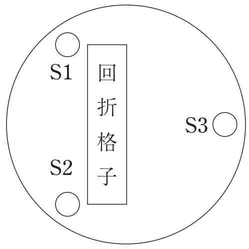



**Fig 6.8** Level adjustment screws of the stage

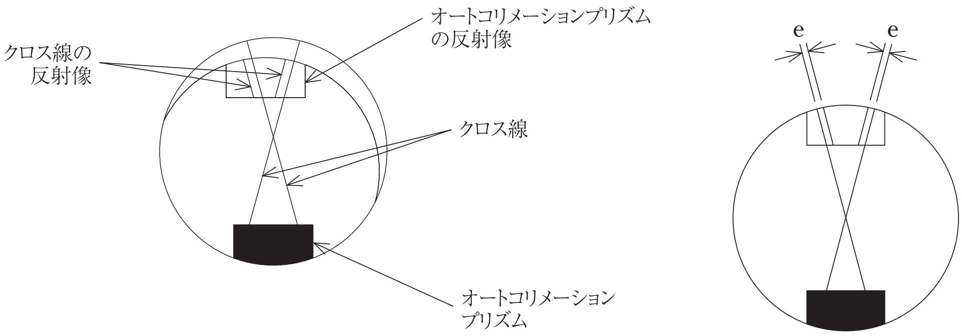



**Fig 6.9** The cross line and its reflected image

(see footnote 1), so that left and right gaps e become the same. Turn the screw S3 of the stage (Figure 6.8) until the gap e is roughly 0.5 to 1 mm.

(2) Turn the stage 180 degrees to inspect the gap e for the back side of the diffraction grating.

(3) If the gap e for the back side is almost the same as in 1, then you are sure you have successfully made the axes perpendicular to each other. If not, repeat 1 and 2 until you get it.

(4) **Turn off the auto-collimation light by turning the housing anti-clockwise (from the top).**

(C) Adjustment of the Collimator

(1) Turn the stage 180 degrees so that the grating faces the telemeter again.

(2) Turn on the Na lamp, and place it in front of the collimator slit.

(3) Move the telemeter until the slit image of the transmitted light is at the center of the eyepiece vision field. Narrow the slit reasonably.

(4) Turn the focus adjustment handle ⑬ of the collimator so that the slit image is focused. This is the zeroth order fringe of the diffraction pattern. Rotate the telemeter, and you will see the images of the higher order fringes one by one.

(5) Adjust the slit width until you can clearly distinguish the $D_1$ and $D_2$ lines in the 4th order diffraction fringe. Then, line up the cross line with the zeroth order fringe, and reset the digital counter to $0°00'00''$.

(6) Turn on the auto-collimation light once again. Rotate the stage until the image of the prism lines up with the center of the slit image (in the left and right direction). Turn off the auto-collimation light once you are done.

(7) If the above adjustments are performed correctly, you should be able to see 8 diffraction fringes for each side. So make sure you can identify these 8 fringes at this point. If now the diffraction angles of the 8th order fringes from both sides are not differing from each other for more than 1 degree, then you can claim victory and put an end to the long adjustments.

**c) Analysis Tasks for Report** After the adjustments, rotate the telemeter to measure the positions (diffraction angles) of the fringes from 1st to 8th order using the digital counter. Make the measurements for both the left and right hand sides, take the average value for each order to determine the diffraction angles $\theta_1$,$\theta_2$,..., then use Equation 6.2 to find the wavelength $\lambda$.

(1) The D-lines are blended into one line for lower order fringes. In this case, the wavelength you get is the average of those of $D_1$ and $D_2$. Moving to the higher orders, $D_1$ and $D_2$ start to become distinguishable. Starting from the order where they can first be distinguished clearly, make the measurements for their positions separately for each order.

Use a format as described below to record and organize your measurements:

<table>
<tr><th colspan="2">$m$</th><th>angle</th><th>$2\theta_m$</th><th>$\theta_m$</th><th>$\sin\theta_m$</th><th>$\lambda$</th><th>$\Delta\lambda_m$</th></tr>
<tr><td>0</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">1</td><td>L</td><td></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>R</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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<tr><td>$D_1$</td><td>R</td><td></td></tr>
<tr><td>8</td><td>L</td><td></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>$D_2$</td><td>R</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

(2) In this experiment, for convenience we simply apply the "slit" number per unit length $n$ as specified on the grating we use. However, the value of n actually changes with temperature to a certain extent. As a result, in a more rigorous sense, one should perform a measurement of $n$ itself first by using lights of known wavelengths and applying Equation 6.2. Only after that can one make measurements of unknown wavelengths using this $n$ obtained experimentally.

(3) The angular precision of this spectrometer is up to 1/2 minute (i.e., 0.5'). From this, one can estimate the maximum error $\Delta\lambda_{\mathrm{m}}$ of wavelength $\lambda$. From the logarithmic differentiation of Equation 6.2, we can calculate the error $\Delta\lambda_{\mathrm{m}}$ (solely from the measurement errors on $\theta$) as

$$|\Delta\lambda_{\mathrm{m}}/\lambda| = |\Delta\theta/\tan\theta_{\mathrm{m}}| \tag{6.4}$$

The error can then be estimated for each order by substituting $\lambda$ and $\theta_{\mathrm{m}}$ into the equation. Here, $\Delta\theta = 1/4'$ ($=7.3\times10^{-5}$rad). For example, in the case of $m=1$, $\Delta\lambda_1 \approx 0.36$nm.

(4) Plot a graph of $\lambda \pm \Delta\lambda_{\mathrm{m}}$ against order $m$ like Figure 6.10. From the result, find $\lambda$ for $D_1$ and $D_2$ as averaged values. Then compare them with those in literature, and discuss any difference between them.

(5) In the analysis above, we simply took the average of the wavelengths from the measurement for each order. However, it is clear from Equation 6.4 that the precision of our measurements varies with the order $m$. In the case where the uncertainty is different among the experimental data points, it is important when we perform the averaging that we put more 'weight' on those data which are more precise than the others . In other words, one should put a weight $w$ on each data point, and take a 'weighted average' of the data. Let $\lambda_1,\ \lambda_2,\ \lambda_3,\ \cdots$ be our experimental data points, with their corresponding weights $w_1,\ w_2,\ w_3,\ \cdots$, then the weighted average $\lambda_{\mathrm{av}}$ is given by

$$\lambda_{\mathrm{av}} = (w_1\lambda_1 + w_2\lambda_2 + w_3\lambda_3 + \cdots)\,/\,(w_1 + w_2 + w_3 + \cdots) = \sum_i w_{\mathrm{i}}\lambda_{\mathrm{i}} / \sum_i w_{\mathrm{i}} \tag{6.5}$$

For each order $m$, let us assume that $w_{\mathrm{m}} = 1/(\Delta\lambda_{\mathrm{m}})^2$. Based on Equation 6.5, try to calculate the weighted averages $\lambda_{\mathrm{av}}$ of $D_1$ and $D_2$ (using the orders where the two lines can be distinguished).

**Attention**

1. This experiment does not have to be repeated by each lab member. Do it collabora-

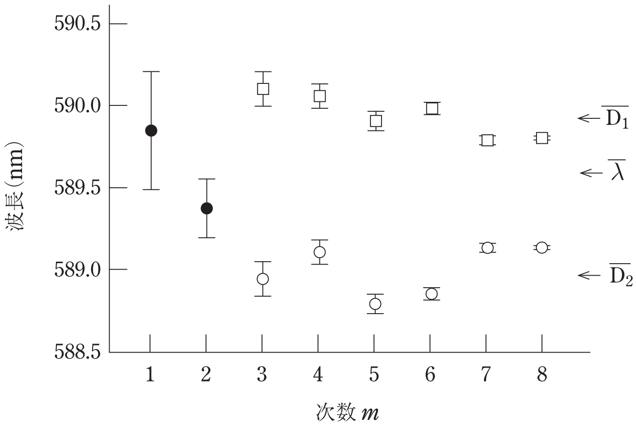



**Fig 6.10** Sample plot

tively.

2. You do not have to draw a schematic diagram for the experimental setup in your report.

〔**Footnote 1**〕 When performing a fine adjustment for the rotation of the telemeter, tighten the clamp ③ first, and then turn the fine adjustment handle ④. **After the fine adjustment is done, make sure you loosen the clamp**. Do not ever try to rotate the telemeter with the clamp tightened.

# Experiment 7. Measurement of Atomic Spectrum with a Prism Spectroscope

## §1 Experimental Objective

This experiment observes atomic spectra by using the historical and most simple optical prism for spectroscopic decomposition. We can get a deep understanding of atomic structure by analyzing the atomic spectrum of hydrogen atoms.

## §2 Overview

Ever since Newton used a glass prism to observe the spectrum of sunlight, spectroscopy has played an extremely important role in the development of modern physics. Observations of various atomic spectra played an especially decisive role in the process leading to the birth of quantum mechanics.

If the incident light on one refractive surface AB of a glass prism is denoted by I (incident angle $i$) and the transmitted light from the other refractive surface is denoted by T as in Figure 7.1, then the deviation angle $\delta$ formed between I and T is determined by the incident angle $i$ and the refractive index. Since this deviation angle changes depending on the difference of the refractive index of glass to the wavelength of light, the transmitted light is decomposed according to wavelength. This is called **dispersion**. If the incident light consists of light of various wavelengths, a spectrum distributed according to their deviation angles is obtained on the transmitted light side. The farther the observation location is from the prism, the more the spectrum will spread spatially. Therefore, the spectral resolution will increase.

## §3 Measurement Principles

### 1. Dispersion Curve

Normally, with a glass prism for observing visible light, the relationship between the wavelength of light and deviation is indicated by a curve like the one shown in Figure 7.2. This kind of curve is called a **dispersion curve**. The horizontal axis has an arbitrary

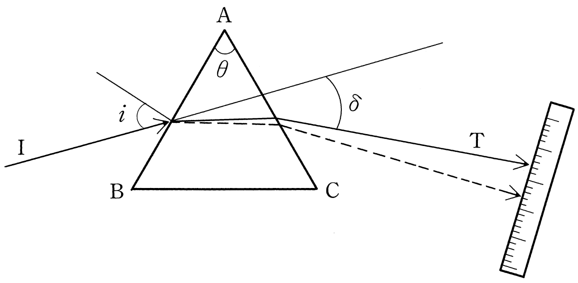
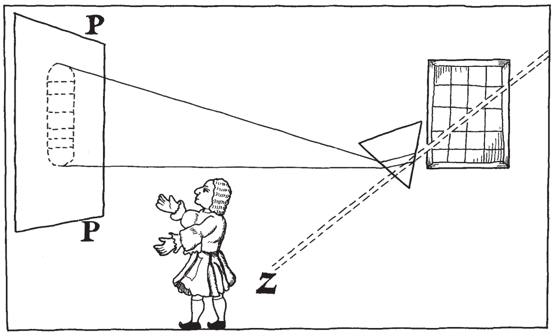



**Fig 7.1** Dispersion of Light by a Prism




Newton's discovery of dispersion.
(Condon & Shortley "The Theory of Atomic Spectra")

scale that is proportional to the deviation angle. If the angle of incidence of the light ray to the prism and the position of the observation scale are fixed, this dispersion curve is characteristic to the spectroscope. Therefore, if the dispersion curve of the spectroscope to be used is known in advance, unknown wavelengths of light can be determined and the spectral distribution can be known. This experiment uses gas light sources of known atomic spectrum to create the dispersion curve of the spectroscope in advance. We then use this information to measure the atomic spectrum wavelengths of a hydrogen gas lamp.

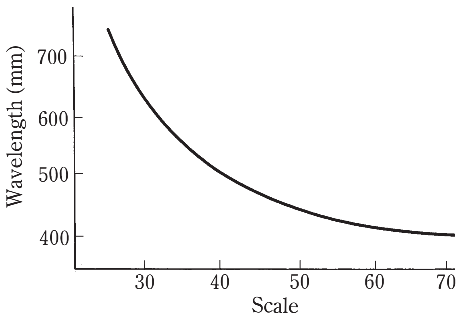



**Fig 7.2** Relationship Between Wavelength and Scale (dispersion curve)

**2. Spectrum of the Hydrogen Atom**

Here, we provide a brief description about the spectrum of hydrogen atoms. This knowledge is necessary to perform the analysis.

When independent atoms flying freely around emit light during the electrical discharge of a gas, their spectrum has an arrangement that is characteristic of the atom type. This atomic spectrum has played an extremely important role in history for elucidating the atomic structure since it directly reflects the motion of electrons within the atom. Johann Jakob Balmer discovered in 1885 that in hydrogen, which is the simplest atom, the regularity seen in the arrangement of spectral lines in the visible part of the spectrum can be accurately represented by the following empirical formula (B is a constant):

$$\lambda = B\frac{n^2}{n^2-4} \qquad (n = 3, 4, 5, ...) \tag{7.1}$$

Later, it was found that there are also many spectral lines in the ultraviolet and infrared parts of the hydrogen spectrum in addition to this visible part, and that they all could be represented by the following formula:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{l^2} - \frac{1}{n^2}\right) \qquad (l < n : \text{natural number}) \tag{7.2}$$

$R_{\mathrm{H}}$ is called the Rydberg constant for hydrogen.

The spectrum is divided into several spectral series according to the value of $l$:

Ultraviolet..........Lyman series ($l = 1, n = 2, 3, 4, \cdots$)

Visible..........Balmer series ($l = 2, n = 3, 4, 5, \cdots$)

Infrared..........Paschen series ($l = 3, n = 4, 5, 6, \cdots$)

The empirical law established in this way was brilliantly explained later by Bohr (1913) in his development of the theory that began with the quantum hypothesis (also refer to the description of the Frank and Hertz experiment). According to this theory, the permitted circular orbits of electrons (radius $r$) must meet the quantum condition of Bohr:

$$2\pi m_{\mathrm{e}}vr = nh \qquad (n = 1, 2, 3, \cdots) \tag{7.3}$$

According to the quantum hypothesis of Bohr, $R_{\mathrm{H}}$ is

$$R_{\mathrm{H}} = \frac{\mu e^4}{8{\epsilon_0}^2ch^3}, \text{where} \quad \frac{1}{\mu} = \frac{1}{m_{\mathrm{e}}} + \frac{1}{m_{\mathrm{p}}} \tag{7.4}$$

($m_{\mathrm{e}}$: mass of electron, $m_{\mathrm{p}}$: mass of proton, e: charge of electron, c: speed of light, $\epsilon_0$: permittivity of free space (electric constant). This theory shows an astonishing match with the experimentally observed result of $R_{\mathrm{H}} = 1.09678 \times 10^7 m^{-1}$).

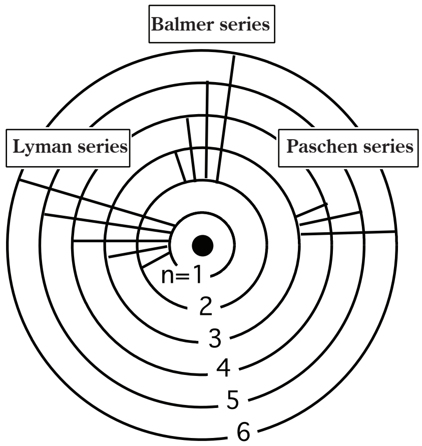



**Fig 7.3** Spectrum of Hydrogen Atom (Schematic Diagram)

## §4 Experimental Apparatus

### 1. Spectroscope (Shimadzu spectrometer model KB II)

The main parts of the spectroscope consist of a collimator C, prism P, telescope T, and scaled tube B as shown in Figure 7.5. The scaled tube has a scale mark S′ on the end. This can be used to adjust the inclination. If scale S′ is illuminated by a lamp, the scaled tube forms an image of the scale as parallel rays, which are reflected by the prism face and enter into the field of view of the telescope.

Rays from the light source to be measured pass through the slit S, become parallel through the collimator, are dispersed by the prism, and finally enter the field of view of the telescope.

With the telescope T adjusted to infinity, it is possible to read the position of the spectral lines on the scale.

**[Caution]**

1 The collimator slit is the heart of the apparatus. Do not damage the sharp edges by over-tightening the slit ”screw” or handling the slit roughly.

2 The prism is adjusted to the ”minimum deviation position” and fixed. (Note 1). Students may remove the center black cover and look inside from the top. However, do not touch the prism with your hand.

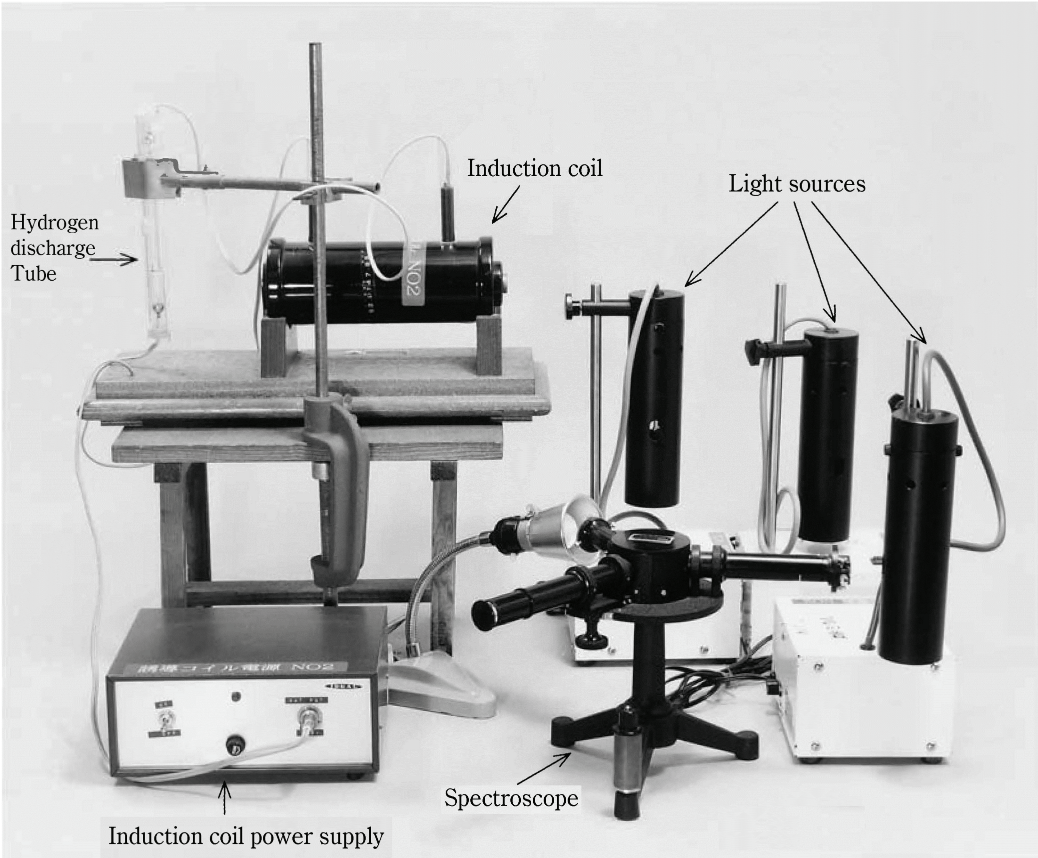



**Fig 7.4** Equipment for Atomic Spectrum Experiment

### 2. Light Source

Hot-cathode discharge tubes enclosing Na, Hg, and Cd are used as the light sources of known wavelength for obtaining the dispersion curve. To light these three types of lamps, set the voltage selection switch to 100 V, turn the power switch to ON, and either hold down the activation pushbutton with your hand or toggle the spring-back switch to one side. A buzzing sound can be heard when the lamp is lit up. If you remove your hand at that time, the lamp will keep on emitting light. After the light intensity has increased over several minutes, it can be used.

## §5 Measurement

### 1. Adjustment of the Spectroscope

**a) Adjustment of Telescope** Illuminate the collimator slit with the sodium (Na) lamp and look through telescope T. Narrow the width of the slit and adjust the length of

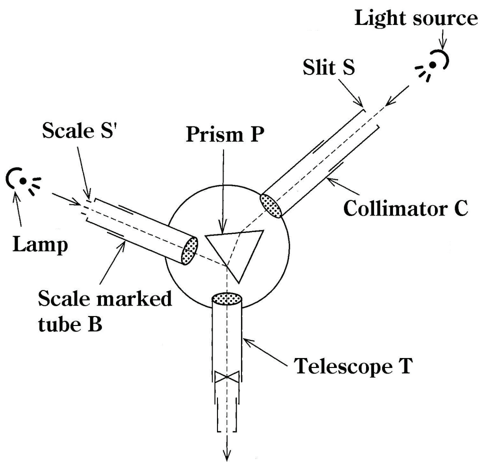



**Fig 7.5** Conceptual Diagram of the Spectrograph

the telescope so that the "spectrum lines can be seen comfortably, precisely, and clearly." The telescope is then adjusted to infinity.

**b) Adjustment of Collimator** This is already adjusted in a fixed way. It is used with the inner tube that is equipped with slit S sufficiently inserted into the outer tube. Although the length of the tube and inclination of slit S (used vertically) can be changed by the attached screw, students must not change them. Although the slit width can be changed by the "screw," be careful not to break it by tightening it too hard.

**c) Adjustment of Scaled tube** Since this is already adjusted in this experiment, you should not touch it. Put the small light sufficiently close to end scale S′ and turn the telescope, which was adjusted to infinity, to look at it. If S′ appears slanted, is too high or too low, or the entire scale cannot be viewed even when the telescope is turned, report this to the instructor. Note that the scale control position is set so that the D line (intense yellow line) of the sodium (Na) lamp or the intense yellow line (579.1, 577.0 nm) of the mercury (Hg) lamp appear at approximately 2.0 to 2.5 on the scale.

## 2. Creating a Dispersion Curve

For the spectral lines of known wavelength, sequentially illuminate the collimator slit using sodium (Na), mercury (Hg), and cadmium (Cd) light sources and read the positions of the spectral lines that appear in the field of view through the telescope (that is, read the deviation) from the scale. On millimeter graph paper, take wavelength for the vertical axis and deviation for the horizontal axis and draw the dispersion curve for the spectroscope that was used. Since the curvature is rather large for this curve, you must obtain observed values for as many known wavelengths as possible to draw an accurate curve.

**[Caution]**

1. Although many spectral lines are visible, you do not know which line corresponds to which wavelength. However, the following lines are extremely intense:

   Na:
   Yellow line 589.6,589.0nm (D line)

   Hg:
   Yellow line 579.1nm Yellow line 577.0nm (2 are visible)
   Yellow Green line 546.1nm
   Blue line 435.8nm

   Cd:
   Red line 643.8nm (standard wavelength)
   Yellow Green lines 508.6nm

   Assume that strong lines like these are for those wavelengths and plot them on the graph paper in these locations. They should roughly lie on a single curve.
2. For other spectral lines, look carefully at the color of the line, brightness, spacing with neighboring lines, and other factors and check the spectrum table in Table 7.1 to estimate the wavelengths of the lines and draw them on the graph. If those points fit well on that curve, those estimates were correct. Enter all lines on the graph in this way and draw one smooth dispersion curve. Use separate symbols (such as o, ×, Δ, etc.) to indicate points for the three lamps so that the values for each lamp can be distinguished.
3. The lines of the spectrum table for the most part can be observed. To do so, you must try various schemes such as widening or narrowing slit S or turning on and off the light illuminating the scale marked tube. You must also be careful since lines that are not in the spectrum table may also be visible.
4. The dispersion curve is always a smooth curve. If the observed points seem to be scattered, the spectrograph may have been poorly adjusted, there may have been

parallax error, or spectral lines may have been taken incorrectly.

5. Since the resolution of the device is about 1.5 nm, spectral lines that are closer than this are observed as a single line. If sometimes a spectral line which is not in the table is observed, it may be ignored when plotting the dispersion curve.
6. Do not just draw the dispersion curve in the report, but also accompany it with a table containing the scale S′ reading, color, and corresponding wavelength for each spectral line that was seen.

### 3. Measurement of Wavelength for Hydrogen Spectrum

Use the dispersion curve you obtained in the previous section to determine the wavelengths of an unknown spectrum. There is a certain degree of latitude in the dispersion curve corresponding to the scattering of the data. Therefore, you can apply the margin $\Delta\lambda$ to evaluate the wavelength $\lambda + \Delta\lambda$ for the observed deviation. This experiment obtains the wavelengths of the spectral lines obtained for a hydrogen discharge tube (a Geissler discharge tube using hydrogen as the gas).

**[Caution]**

1. Lighting the hydrogen discharge tube. Since hydrogen gas has an especially high discharge voltage compared with other gases, an induction coil is used to apply a high voltage to cause the electrical discharge. The primary winding of the induction coil is operated by a pulse voltage using a thyristor. Since the secondary winding of the induction coil produces a high voltage amounting to several dozen kV even though the current is small, be careful not get too close to it with your hands.
2. Besides three distinct spectral lines, one more faint spectral line can be observed for the hydrogen spectrum at the violet end. However, to observe this line, you must try various schemes such as widening or narrowing slit S or turning on and off the light illuminating the scale marked tube. You must also be careful since lines that are not in the hydrogen spectrum table may also be visible.

## §6 Analysis Tasks for Report

1. Obtain the Rydberg constant $R_{\mathrm{H}}$ by using all of the wavelength data for hydrogen obtained in this experiment. Draw a graph with $1/n^2$(n=2,3,4,...) for the horizontal axis and $1/\lambda$ for the vertical axis. The Ballmer series spectrum lies on a rightward descending straight line that passes through the coordinate $(1/2^2,0)$. If the values of n = 3, 4, 5, ... are assigned for each observed value $\lambda$ of the hydrogen spectrum and the

**Table 7.1** Spectrum Table (brackets enclose single lines, circles indicate intense lines)

| Sodium (Na) | | Mercury (Hg) | | Cadmium (Cd) | |
|---|---|---|---|---|---|
| $\lambda$ (nm) | Color | $\lambda$ (nm) | Color | $\lambda$ (nm) | Color |
| 616.1<br>615.4 | Narrow red | 690.7<br>671.6 | Light red | 738.5 ⎫<br>738.4 ⎭<br>734.6 | Red |
| ◎589.6 ⎫<br>◎ 589.0 ⎭ | Intense yellow<br>(D line) | 589.0 ⎫<br>588.9 ⎭ | Intense yellow | 646.5<br>◎643.8 | Heavy Red (standard) |
| 568.8 ⎫<br>568.3 ⎭ | Yellow green | 580.4<br>◎579.1 | Intense yellow | 636.0<br>632.5 | Narrow red |
| 515.4 ⎫<br>514.9 ⎭ | Narrow green | ◎577.0 | Intense yellow | 611.2 ⎫<br>609.9 ⎭ | Orange |
| 498.3 ⎫<br>497.9 ⎭ | Intense narrow blue | ◎546.1 | Intense yellow green | 563.7 | Narrow green |
| 475.2 ⎫<br>474.8 ⎭ | Pale violet | 491.6 | Narrow dark green | 515.5 | Pale green |
| | | | | ◎508.6 | Intense blue green |
| 466.9 ⎫<br>466.5 ⎭ | Pale violet | ◎ 435.8<br>◎ 434.8 | Intense blue | ◎480.0 | Slightly intense blue green |
| 439.3 | Pale violet | ◎407.8<br>◎404.7 | Narrow violet | ◎467.8 ⎫<br>◎466.2 ⎭<br>441.5 | Intense blue<br>Intense narrow violet |

points ($1/n^2$,$1/\lambda$) are entered on the graph, the n for which the points are accumulated overall on the straight line passing through ($1/2^2$,0) can be selected for each $\lambda$. Mark this selected group of points larger to illustrate the regression line passing through ($1/2^2$,0). Add straight lines with minimum and maximum slope according to the scattering of this group of points.

2. The slope of this regression line is equal to $-R_{\mathrm{H}}$ and the vertical axis intercept at $1/n^2 = 0$ is equal to $R_{\mathrm{H}}/4$. Evaluate the value of $R_{\mathrm{H}}$ from this relationship including an observation margin as $R_{\mathrm{H}} \pm \Delta R_{\mathrm{H}}$. Then, compare it with the reference value in the note below.
3. Ionization Energy of Hydrogen Atom

   $R_{\mathrm{H}}$ multiplied by $hc$, i.e. $hcR_{\mathrm{H}}$, has the dimension of energy. It represents the ionization energy of hydrogen in the ground state. (This corresponds to the case where $l = 1, n = \infty$ in 7.2). Try to calculate the ionization energy in units of eV from the value of $R_{\mathrm{H}}$ (reference value: 13.6eV).
4. Evaluation of the Size of Hydrogen Atom

   Using $R_{\mathrm{H}}$, let's estimate the size of an hydrogen atom. The energy $E$ of electrons performing circular motions of radius $r$ under the influence of the Coulomb force emanating from the nucleus can be represented as:

$$E = \frac{m_{\mathrm{e}}v^2}{2} - \frac{e^2}{4\pi\epsilon_0 r} \tag{7.4}$$

   Since the Coulomb force is a centripetal force, it follows that:

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m_{\mathrm{e}}v^2}{r} \tag{7.4}$$

   If we get rid of $v$ from (4) by using (4), the energy of the electron becomes:

$$E = -\frac{e^2}{8\pi\epsilon_0 r} \tag{7.4}$$

   Since the energy of the electron in the ground state is $-hcR_{\mathrm{H}}$, by setting this quantity equal to (4), we find the orbital radius of the ground state electron is given by the following equation:

$$r = \frac{e^2}{8\pi\epsilon_0 hcR_{\mathrm{H}}} \tag{7.4}$$

   Try to get $r$ from the value of $R_{\mathrm{H}}$. This value of $r$ is called the Bohr radius $a_{\mathrm{B}}$, which is a measure of the size of the atom. (reference value $a_{\mathrm{B}} = 5.29 \times 10^{-11}$ m)

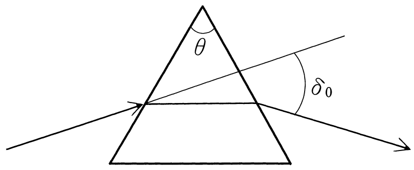



**Fig 7.6** Minimum Deviation Angle by the Prism

## §7 Further Study

**[Note 1] Minimum Deviation Angle** The deviation angle $\delta$ in Figure 7.1 varies according to the angle of incidence $i$ of the incident light. When $i$ is changed, it is at the position where $\delta$ is the minimum. It is apparent according to a simple geometric consideration that this is when the light ray passes symmetrically through the prism (Figure 7.6). The deviation at this time is called the "minimum deviation angle."

The minimum deviation angle $\delta_0$ and the refractive index $n$ of the prism are related as follows:

$$n = \sin\left(\frac{\delta_0 + \theta}{2}\right) \Big/ \sin\left(\frac{\theta}{2}\right) \tag{7.5}$$

where $\theta$ is the vertex angle of the prism.

In a spectrometer that uses a glass prism to observe normal visible light, fix the collimator and prism so that they are at the minimum deviation angle for the D line of sodium (Na), for example, to observe a small deviation angle distribution centered on this. This setup is because all wavelengths are almost at the center of the visible region and the light is intense.

**[Note 2] Refractive Index and Wavelength** The relationship between the refractive index $n$ of glass and the wavelength of light can be approximated in the following form:

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots \tag{7.6}$$

Although this was first discovered empirically (Cauchy′s equation), it was verified later by classical dispersion theory.

The dispersion curve $\delta$ is not strictly proportional to the refractive index $n$, (Note 1, formula (7.5)). Since the $\delta$ changes in the small ranges, the dispersion curve will reflect the wavelength-dependence of $n$. So, we illustrate the deviation as a function $1/\lambda^2$ in accordance with the above discussion. In the region where $1/\lambda^2$ is small, the figure is close to a straight line, and hence it is expected that more accurate results can be obtained. Try to obtain the wavelength from this figure.

According to dispersion theory, when a material is considered to consist of a homogenous collection of oscillators its refractive index $n$ and the light frequency $\omega$ are related as follows:

$$n^2 = A' + \frac{B'}{\omega_0^2 - \omega^2} \tag{7.7}$$

In this formula, $\omega_0$ is the resonance frequency of the oscillators and A' and B' are constants. In the region where $\omega \ll \omega_0$ (that is away from resonance), if $\omega$ is transformed to the wavelength and expanded, equation 7.6 above is obtained.

# Experiment 8. Franck-Hertz Experiment

## §1 Experimental Objective

The objectives of this experiment are to verify that atoms have discrete energy states using the inelastic collisions of electrons and neon (Ne) atoms as well as to obtain the lowest excitation energy of the neon atom.

## §2 Overview

When classical electromagnetism is applied to a simple atomic model in which electrons orbit the nucleus in a circular motion, the electrons are found to emit electromagnetic waves with the same frequency as their rotation. As a result, they lose energy and eventually fall into the nucleus, meaning the atom itself cannot exist in a stable state. Further, since the electrons are losing energy the frequency of the emitted electromagnetic radiation should have a continuous spectrum, which contradicts the observation that the emitted light possesses a definite frequency.

To address these issues Bohr made the following quantum mechanical hypotheses concerning the states of electrons in an atom (1912).

(1) An electron within an atom may only exist in states with discrete energies. The energies of these states depend on the motion of the electron and they do not vary with time (they are steady state). An energy change occurs only when the electron transitions from one steady state to another.

(2) Light (an electromagnetic wave) that is emitted or absorbed when the electron transitions from one steady state with energy $E_m$ to another steady state with energy $E_n$ is monochromatic, and its frequency $\nu$ satisfies the condition,

$$h\nu = E_m - E_n, \tag{8.1}$$

where $h$ is Planck's constant.

Bohr further imposed constraints on the allowed values of the electron's angular momentum and he successfully demonstrated that the hydrogen atom in fact possesses discrete energy levels. Though the Bohr quantum hypotheses cannot explain why no radiation is emitted even when the electrons undergo acceleration, such as circular motion, they beautifully reproduce the observed emission spectrum of the hydrogen atom and thereby opened the first era of quantum mechanics.

The Bohr quantum hypotheses were directly verified experimentally by Franck and Hertz in 1913. In their experiment they used an electron beam instead of electromagnetic waves to cause electrons in atoms to change states. When electrons collide with an atom with more kinetic energy than its excitation energy, part of their energy is lost to exciting the atom. The energies of electrons used in the experiment were represented in units of electron volts, eV, where e is the elementary electric charge (C, measured in Coulombs) and V is the accelerating voltage (V, measured in Volts). Let $E_0$ denote the ground state energy of the atom and let $E_1$ denote the energy of the first excited state above this ground state. Each of these energies can be expressed in units of electron volts as $eV_0$ and $eV_1$, respectively. If $eV_E$ denotes the lowest excitation energy of the atom, then based on Equation (8.1),

$$V_E = V_1 - V_0. \tag{8.2}$$

Though the Bohr hypotheses successfully explain the energy levels of the hydrogen atom with its single electron, in this experiment, you will obtain $V_E$ for neon, which has multiple electrons. For this reason, calculating the energy levels of neon is difficult. In fact the energy difference between the ground state and the first excited state have not been calculated exactly. It has instead been estimated using various approximations. The lowest excitation voltage that has been obtained empirically is $V_E = 16.7\ V$ and the next lowest is $18.7\ V$.

## §3 Measurement Principle

In this experiment the energy loss of an electron beam exciting Ne atoms will be measured and the subsequent Ne excitation light will be observed.

**(1) Neon Excitation from Inelastic Collisions with an Electron Beam**

As shown in Figure 8.1, inside a four electrode vacuum tube (tetrode) filled with neon gas electrons from cathode K are accelerated between grids $G_1$ and $G_2$. When the accelerating voltage $V_{acc}$ is increased from $0\ V$ the electrons will reach the plate (anode) P without losing any energy even if they collide with Ne atoms on the way. Under these conditions the current at the anode will increase monotonically as determined by the voltage-current characteristics of the vacuum tube (see the Thermionic Emission experiment). However, since a retarding (also called a "reverse" or "stopping") voltage of approximately $6\ V$ is applied to the plate relative to grid $G_2$, the plate current $I_p$ will begin to increase after the accelerating voltage surpasses this voltage.

Increasing the accelerating voltage $V_{acc}$ increases the energy of the electrons. When their

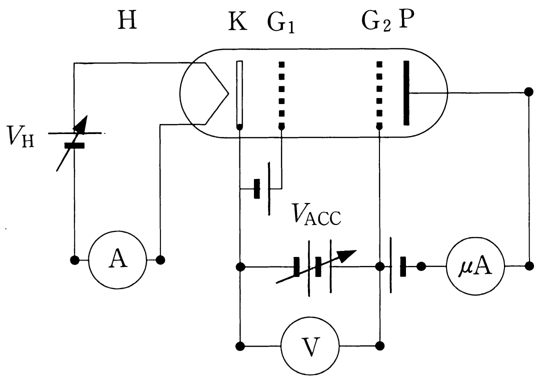



**Fig 8.1** Schematic diagram of the equipment used in the Franck-Hertz experiment.

energy approaches $eV_E$, their collisions with Ne atoms become inelastic and they lose energy by exciting the atom. After these collisions, electrons with less energy than the retarding voltage cannot reach the plate and as a result, $I_p$ decreases suddenly. Note that since thermions are emitted from the cathode with a distribution of initial energies and because an inelastic collision with an atom is not always guaranteed the current may not drop to zero.

If the accelerating voltage is increased further, $I_p$ will increase again when the energy of the electrons exceeds the retarding voltage even after they have lost energy to inelastic collisions. However, if their energy after such collisions exceeds the excitation energy $eV_E$ they may undergo a second inelastic collision before reaching $G_2$ and lose energy again. As a result, the plate current will once again drop. If this process is repeated the plate current will have maxima each time $V_{acc}$ increases by $V_E$ as shown in Figure 8.2.

**(2) Observation of Excitation Luminescence**

Neon atoms excited by inelastic collisions with electrons will transition to lower energy states by the emission of light, repeatedly if necessary, until they finally reach their lowest possible energy state. The frequency of the emitted light follows the second Bohr hypothesis and if it is in the visible spectrum the light can be observed with the naked eye. Neon's ground state electron configuration has two electrons each in its $1s$ and $2s$ orbitals and 6 electrons in its $2p$ orbital. This configuration is known as a "closed shell" ($1s^2 2s^2 2p^6$). The minimum excitation energy corresponds to the energy needed to move one electron from the outermost $2p$ orbital to the $3s$ orbital. For neon this energy is 16.7 eV which corresponds to ultraviolet light and is therefore not in the visible spectrum. However, in this experiment excitation light in the visible spectrum (mostly red) can also be observed. As shown in Figure 8.3, neon has many energy levels. Analyzing its excitation spectrum yields several

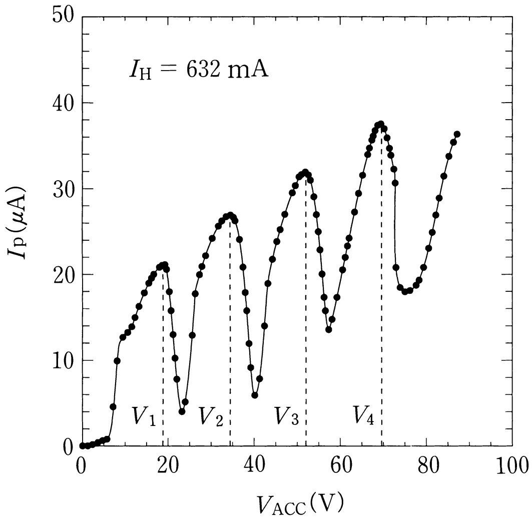



**Fig 8.2** Example result from the Franck-Hertz experiment.

transitions that produce visible light, but among those the one with the most intensity has a wavelength of 585.2488 nm. This corresponds to the transition of an outer electron from the $3p$ to $3s$ orbital and has an energy of 2.11 eV. When such light is observed it indicates that the original electrons had enough energy to cause excitations from the $2p$ to $3p$ state (18.7 eV), or they further excited atoms that were already in the $3s$ state to $3p$. In either case the $3p \rightarrow 3s$ transition produces the observed light. It should be noted that the observed excitation light depends on the density and energy of the incident electrons, the density of the Ne gas in the tetrode, and on other factors such as the distance between the tetrode's electrodes.

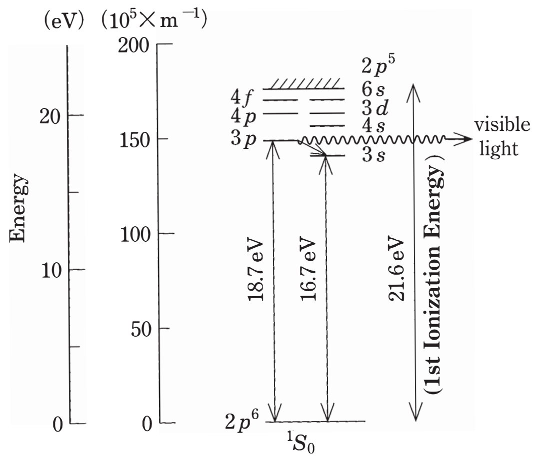



**Fig 8.3** Energy levels of the neutral Ne atom.

## §4 Equipment

In this experiment you will use the following equipment (see Figure 8.4).

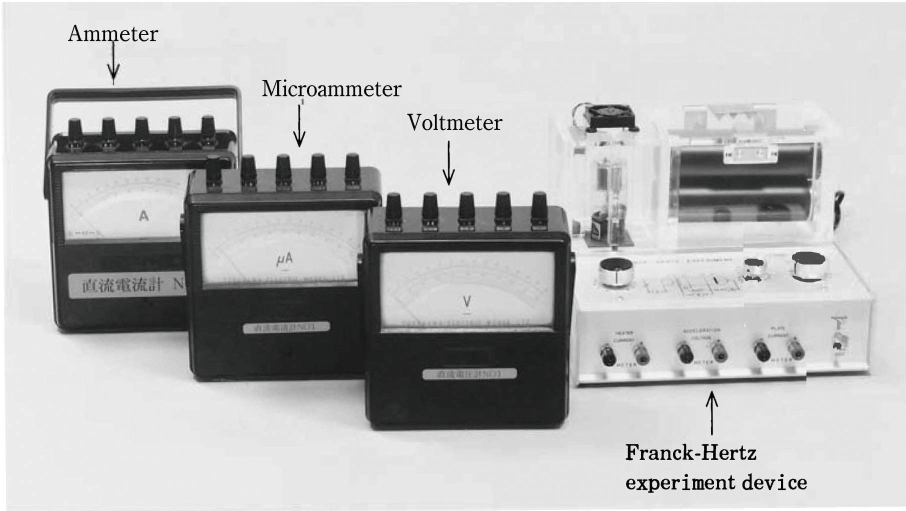



**Fig 8.4** Franck-Hertz experimental setup

(a) Franck-Hertz experiment device (A collaborative production by Kyoto University and Eclai Co., Ltd.)

(b) DC voltmeter (0 V to 100 V) for measuring the accelerating potential (abbreviated as V-meter below)

(c) DC ammeter (0 A to 1 A) for measuring the heater current (abbreviated as A-meter below)

(d) DC microammeter (0 to 100 $\mu$A) for measuring the plate current (abbreviated as $\mu$A-meter below)

Figure 8.5 shows the Franck-Hertz experiment device with labels on its main components. The structure of the tetrode is shone in Figure 8.6.

| | |
|---|---|
| POWER | Main power and heater current power switch . |
| Cylindrical Shades (2) | Used when observing excitation luminescence. |
| Tetrode | Four electrode vacuum tube filled with Ne gas. Contains a heater, cylindrical cathode, grid and plate. |
| HEATER CURRENT | Dial for adjusting the heater current. |
| ACCELERATION VOLTAGE | Dial for adjusting the accelerating voltage. |
| PLACE CURRENT ZERO ADJ. | Dial for adjusting the zero point of the plate current. |
| A-METER | Terminals for connecting the DC ammeter to measure the heater current. |
| V-METER | Terminals for connecting the DC voltmeter to measure the accelerating voltage. |
| $\mu$A-METER | Terminals for connecting the DC microammeter to measure the plate current. |

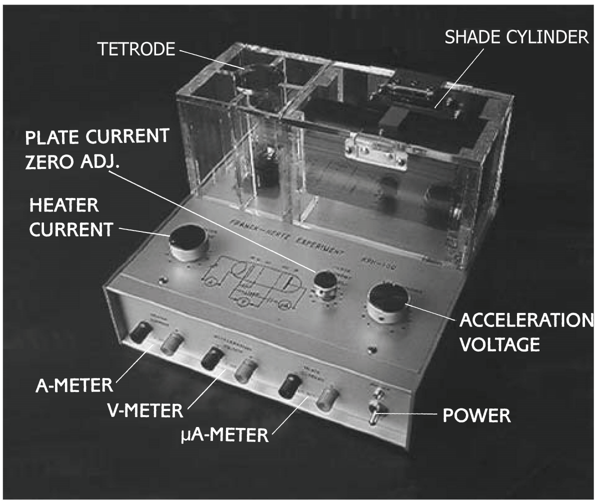



**Fig 8.5** Names of each part of the Franck-Hertz experiment device.

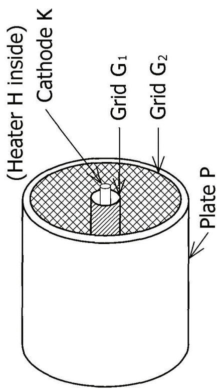



**Fig 8.6** Schematic diagram of the internal structure of a tetrode.

## §5 Method

### Measurement Preparations

(1) Wire the A-meter, $\mu$A-meter, and V-meter, to the front of the Franck-Hertz experiment device to measure the heater current, plate current, and accelerating voltage, respectively. The A-meter should be wired to use its 1 A terminals, the $\mu$A-meter to its 100 $\mu$A terminals, and the V-meter to its 100 V terminals.

(2) Verify that the dials for the heater current and acceleration voltage on the top panel of the Franck-Hertz experiment device are turned all the way to the left. (**Warning** Do not forcibly turn the dials too far right or left at any time; they have an angular range of only about 300°.) Note that during the experiment there is no need to change the plate current meter. It should always be fully turned to the left.

(3) Make sure the tetrode is not covered by the cylindrical shades.

(4) Plug in the power cord of the Franck-Hertz experiment device and turn its POWER switch on. Verify that about 400 mA of current are flowing to the heater. Wait for about one minute for the cathode to warm up and reach a sufficiently high temperature.

(5) Use the zero-point adjustment dial to set the plate current to zero.

(6) Set the accelerating voltage $V_{acc}$ to about 90 V. Next, slowly increase the heater current to so that the plate current $I_p$ is between about 60 and 80 $\mu$A. Typical heater currents are 600 mA to 800 mA. Be careful to adjust the heater current slowly - it will take time for the plate current to respond. Wait until the plate current stops changing, allowing the cathode to warm up again. Once the plate current has stabilized, record the heater current $I_h$. Afterwards return the accelerating voltage to 0 V and adjust the zero point of the $\mu$A-meter again.

**Measurement 1: Measuring the plate current's dependence on the accelerating voltage**

(1) Starting from 0 V slowly increase the accelerating voltage and confirm that the plate current peaks four or five times. Record the accelerating voltages that corresponded to maximum values of the plate current.

(2) Next, gradually increase the accelerating voltage from 0 to 100 V in several steps and record the observed plate current. Plot these values on a graph. In order to see the peaks and valleys in the plate current, voltage steps of about 0.2 to 0.5 V are necessary. Larger steps of 1 V can be used otherwise. Note that above 30 V the voltmeter's scale may be too coarse to accurately measure 0.5 V steps.

(3) Repeat steps (1) and (2) using a slightly different heater current. If you have a partner switch your roles in the experiment. Otherwise repeat the measurement by yourself.

**Measurement 2: Observing excitation luminescence**

(1) Stack the two cylindrical shades together into a single long tube and slide it over the tetrode. Look through the tube down onto the tetrode and change the accelerating voltage to observe patterns produced by the excitation luminescence of neon atoms (Figure 8.7). Verify that rings of visible light appear when the accelerating voltage is about 2 V larger than its value at the peak value of the plate current. Make a note

of how the ring pattern changes with changes in the accelerating voltage. (**Warning** With the shades covering the vacuum tube heat can no longer escape from inside and will cause the tube's temperature to rise. As a result, the plate current will gradually increase. Carry out your observations quickly. Remove the shades as soon as your observations have finished. There is no need to repeat this measurement. )

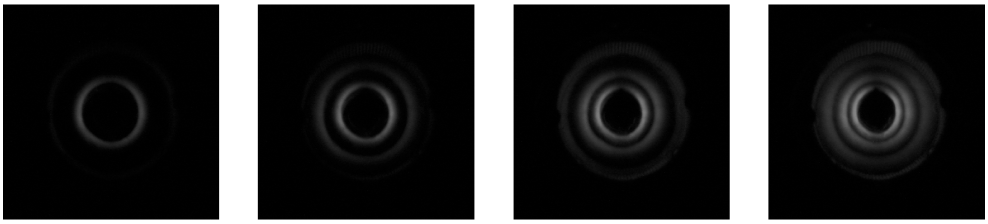


**Fig 8.7** Neon excitation luminescence rings. In this example the accelerating voltages were, from left to right, 40 V, 50 V, 60 V, and 70 V. The actual ring color is red.

(2) When all measurements have finished return the shades to their original position and turn all dials other than the zero adjust dial all the way to the left. Afterwards turn the POWER switch to the OFF position and unplug the power cord. Disconnect all wires and return them to their designated positions.

## §6 Analysis Topics for Report

Follow the instructions below to analyze the data from your measurements.

(1) Plot the relationship between accelerating voltage and plate current and draw a smooth line that connects each of the data points. Plot the results of both of your measurements as two sines on a single graph. Make the graph as easy to read and interpret as possible. For example, consider the size of the data points and the overall appearance of the figure.

(2) Explain why it is reasonable to use the accelerating voltage that corresponds to the maximum observed value of the plate current as the lowest excitation voltage of neon. Would it be possible to use the voltage corresponding to the valley of the plate current for this purpose? Why or why not?

(3) Due to the energy needed to draw thermions out of the cathode and because of effects like the energy distribution of the accelerated electrons, the accelerating voltage that produces the plate current's first maximum does not correspond exactly to the minimum excitation potential of neon. Instead this potential is extracted using the difference of accelerating potentials that produce the plate current's maxima. Read off the voltages

that correspond to the peaks in your measured current curve and order them from smallest to largest as $V_1$,$V_2$, etc.. Then, taking $eV_W$ as the energy needed to extract thermions from the cathode, the following set of equations holds:

$$V_1 = V_E + V_W, \tag{8.3}$$

$$V_2 = 2V_E + V_W, \tag{8.4}$$

$$V_3 = 3V_E + V_W, \tag{8.5}$$

$$V_4 = 4V_E + V_W. \tag{8.6}$$

(8.7)

Accordingly, $V_E$ can be determined as

$$V_E = \frac{1}{2}\left(\frac{1}{2}(V_3 - V_1) + \frac{1}{2}(V_4 - V_2)\right). \tag{8.8}$$

Compute the value of $V_E$ for both of your measurements and compare them with the expected value of neon's minimum excitation energy.

(4) Make a connection between the symmetric construction of the tetrode and the observed excitation luminescence to explain why rings are observed.

(5) Discuss the relationship between the number of excitation luminescence rings and peaks in the plate current. Additionally, explain how and why the rings radii change with changes in the accelerating voltage.

(6) Discuss how the graph of accelerating voltage versus plate current would change if the energy levels in the atom were not discrete.

(7) Discuss how your results depend on the density of the neon in the tetrode if at all.

# Experiment 9. Measurement of Planck's Constant

## §1 Experimental Objective

The purpose of this experiment is to measure the defining constant of quantum mechanics, Planck's constant $h$, using the photoelectric effect.

## §2 Overview

In the latter half of the $19^{th}$ century, the spectrum of light emitted from the hot material inside of blast furnaces (known as thermal radiation) could not be explained using the classical theory of waves. In order to rescue the classical theory Planck introduced his **quantum hypothesis** suggesting that, much in the same way that matter is composed of indivisible atoms, the energy of light is not a continuous quantity but is instead made up of discrete energy elements. Using this hypothesis, he derived a radiation formula, also known as the Planck formula, and showed its predictions agreed well with experimental data. Planck introduced the idea of an **energy quantum** as a discrete energy element with energy $\varepsilon = h\nu$, and hypothesized that light (electromagnetic wave) with a frequency $\nu$ has an energy that is an integer multiple of this quantum. Today $h$ is known as the Planck constant and it carries units of (energy) $\times$ (time).

On the other hand, the photoelectric effect, in which electrons are emitted when light is shined on a metallic surface (Figure 9.1), had been discovered at the end of the $19^{th}$ century. In 1905, Einstein introduced the light quantum hypothesis, suggesting that light is not a wave, but is instead a collection of particles (light quanta or photons) in order to explain Lenard's (1902) studies of the photoelectric effect. Assuming that light of frequency $\nu$ is a quantum of energy, $h\nu$, then if that light is shone on a metallic surface it will impart energy to an electron in the metal. That electron will have a kinetic energy $K$ and may escape from the metal if its energy is greater than the metal's work function. Labelling the work function $W$ and the maximum kinetic energy of such electrons as $K_{\text{max}}$, the following equation holds.

$$K_{\text{max}} = h\nu - W. \tag{9.1}$$

In this way, $h$, which was introduced in Planck's formula, and light quanta became associated with one another. Millikan studied the relationship between $\nu$ and $K_{\max}$ in 1916 to test Equation (9.1) quantitatively and thereby measured $h$.

In this experiment you will use a photocell and a technique known as the "stopping potential" method to measure the value of Planck's constant.

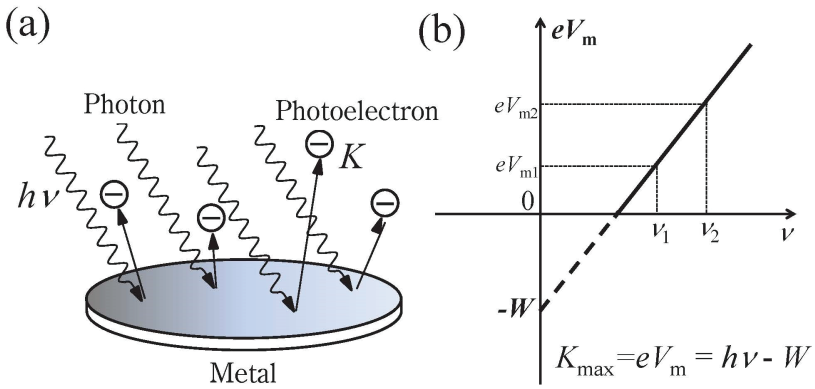



**Fig 9.1** Schematic of the photoelectric effect. Panel (a) shows photoelectrons emitted from a metallic surface. A photon of energy $h\nu$ imparts energy to an electron and it escapes the metal with kinetic energy $K$. Panel (b) shows a graph of the relation $K_{\max} = eV_m = h\nu - W$. The slope of the resulting line can be used to estimate $h$.

## §3 Measurement Principle

Figure 9.2 shows a schematic of the photocell that is used in this experiment. When light of frequency $\nu$ impinges on the photocathode, electrons with kinetic energy $K$ are emitted. If a

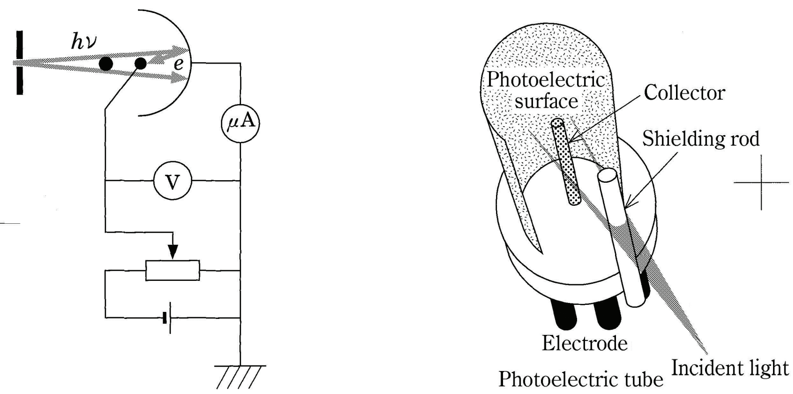



**Fig 9.2** Schematic diagrams of the photoelectron measurement principle and a photocell.

negative voltage, $-V$, called the retarding potential or reverse bias, is applied to the collector relative to the photocathode, then only electrons with $K > eV$ can reach the collector and be observed as a current. Since the electrons are emitted with a maximum kinetic energy given by Equation (9.1), if the voltage is raised to $V_m$, where $eV_m = K_{\text{max}} = h\nu - W$, no electrons reach the collector and the observed current drops to zero. Accordingly, $V_m$ is known as the "stopping" potential. As shown in Figure 9.1(b) if stopping potentials $V_{m1}$ and $V_{m2}$ are measured for two frequencies of light, $\nu_1$ and $\nu_2$, Plank's constant can be determined as $h = e(V_{m1} - V_{m2})/(\nu_1 - \nu_2)$.

However, in an actual experiment a small current will still be observed at the collector even if the reverse bias is raised above $V_m$. This is because the photocathode has a finite temperature and, as shown in the supplements section, the emitted electron energy distribution is distorted by thermal excitation. In this experiment you will estimate the value of Planck's constant using the stopping potential method, but if you consider the effects of finite temperature, using the the asymptotic method explained in the supplemental section will yield a more accurate value.

**Stopping Potential Method**

For sufficiently large reverse biases the photocurrent becomes negligibly small. However, as stated above the current will not become 0 A. For this reason,when the current becomes

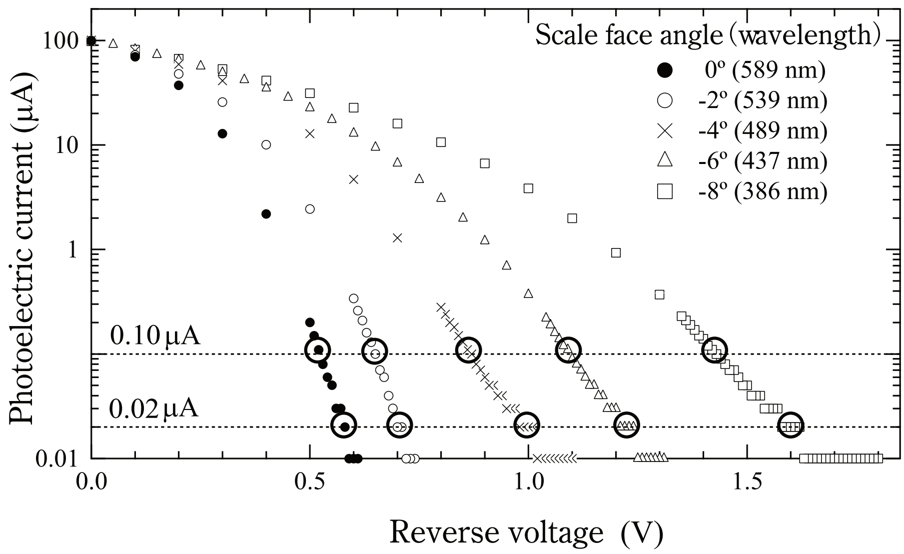



**Fig 9.3** Example of the relationship between photocurrent and reverse bias for 386 nm and 589 nm light. Places where the photocurrent, $I_{\text{photo}}$, is 0.02 (0.10) $\mu$A are chosen to be the experimental zero point of the current and are marked with circles. The reverse bias at each of these points is taken to be $V_s$.

sufficiently small, for example 0.10 $\mu$A or 0.02$\mu$A, the stopping potential $V_s$ can be regarded as corresponding to the maximum electron energy $K_{\text{max}}$. That is, we will take $K_{\text{max}}$ to be $eV_s$ (see Figure 9.3 for example). Measuring $V_s$ for several frequencies of light, $\nu$, and using the relation

$$eV_s = h\nu - W, \tag{9.2}$$

Planck's constant $h$ can be determined.

## §4 Equipment

The equipment used in this experiment are as follows (see also Figure 9.4).

(a) Planck's constant measurement instrument (HA-30)

(b) DC voltmeter/ammeter (KU-1AV, abbreviated as AV-meter below)

Figure 9.5 shows the names of various parts of the Planck's constant measurement instrument (HA-30).

- **Light source:** Halogen lamp (Temperature 3300 K).
- **Photocell:** A semi-cylindrical photoelectric surface (photocathode) with a rod-shaped collector (anode) located at its center. The photocathode surface has been coated with an antimony-cesium alloy (Sb-Cs) which has a work function of $W_{Sb-Cs} \leq 2$eV (Figure 9.1).
- **Spectroscope:** A spectroscope is a device that extracts monochromatic light (light with a single wavelength) from a light source with a variety of wavelengths. It consists of a collimator to focus the input light into parallel rays, a diffraction grating to separate the light into its spectral components, and a telemeter lens to focus the separated light onto a slit on the front of the photocathode. A reflective diffraction grating with lattice constant $d = 1/1200$ mm is used. Table (6.1) shows the relationship between angle of the grating and the wavelength of light (first-order) that it passes.
- **Control panel**

  POWER: Power switch

  LAMP: Halogen lamp switch

  COLLECTOR VOLTAGE: Reverse bias voltage adjustment dial

  ZERO ADJ: Photoelectric current zero-point adjustment dial

  A-METER: DC ammeter connection terminals for measuring photoelectric current

  V-METER: DC voltmeter connection terminals for measuring the reverse bias

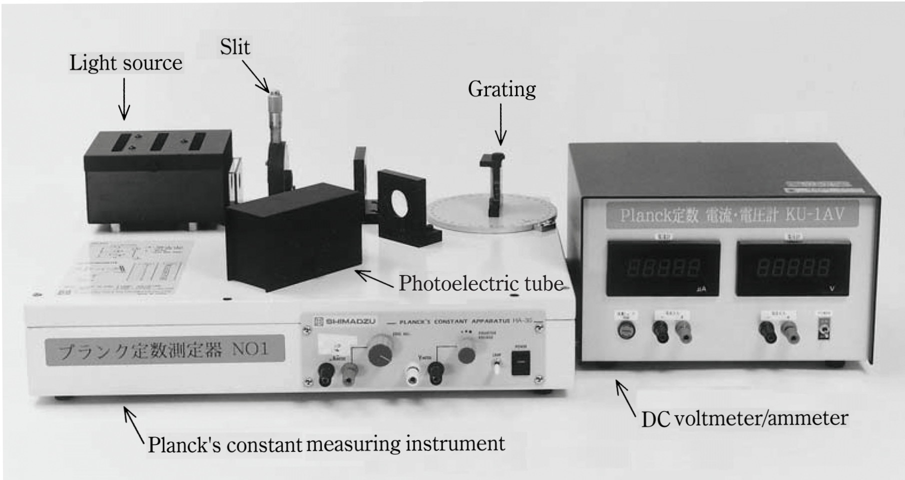



**Fig 9.4** Planck's constant measurement instrument.

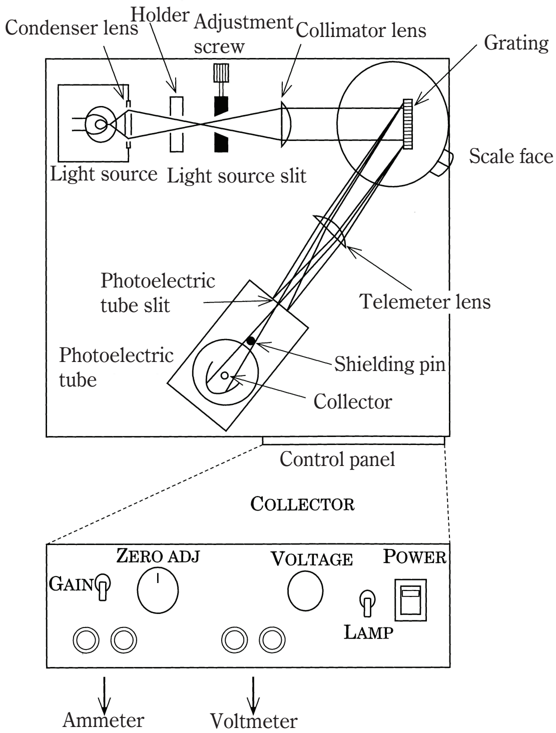



**Fig 9.5** Names of various parts of the Planck's constant measurement instrument.

**Table 9.1: First-order wavelength and frequency of light produced by various angular settings of the diffraction grating.**

| First-order light | | | |
|---|---|---|---|
| Angle (deg) | Wavelength (nm) | Frequency ($\times 10^{14}$ Hz) | Color |
| 2.0 | 639 | 4.70 | Red |
| 1.0 | 614 | 4.88 | Orange |
| 0.0 | 589 | 5.09 | Yellow |
| -1.0 | 564 | 5.31 | |
| -2.0 | 539 | 5.56 | Green |
| -3.0 | 514 | 5.83 | |
| -4.0 | 489 | 6.14 | Blue |
| -5.0 | 463 | 6.47 | |
| -6.0 | 437 | 6.86 | |
| -7.0 | 411 | 7.29 | Violet |
| -8.0 | 386 | 7.78 | Ultra-violet |
| -9.0 | 359 | 8.34 | |

## §5 Method

**Preparations**

(A) Measurement Preparations

1) Verify that the POWER and LAMP switches are OFF.
2) Plug in the device and only turn the LAMP switch ON.
3) Remove the black cover from the spectrometer being careful not to touch other parts of the apparatus. Set the light source slit to a reasonable opening so that you can see light on the grating. Make a note of which way to turn the dial to both open and close the slit.
4) Set the diffraction grating to the 0.0 degree position by aligning the scale to the zero line. In this position 589 nm light will shine on the photocell - it should be yellow.. Verify that a continuous spectrum of light is visible on the photocell's cover by changing the position of the diffraction grating. Afterwards return the black cover to its original position.
5) Turn the LAMP switch OFF and wire the terminals on the front of the instrument panel to the photoelectric current $\mu$A ammeter and reverse bias voltage V-meter.
6) Turn the POWER switch ON and allow the measurement device to warm up for about 20 minutes.
7) At the same time plug in the power for the DC voltmeter, turn its POWER switch ON and allow it warm up for about 20 minutes. After the devices are warmed up, proceed with the instructions (B).

(B) Spectrometer Preparations

1) Turn the COLLECTOR VOLTAGE dial on the upper part of the front panel all the way to the left until the measured voltage is 0.0 V. Turn the dial all the way to the right and verify that the reverse bias voltage is at least 3 V. (**Be careful!** Do not forcibly turn the dial too far to the left or right at any time during the experiment.)
2) Set the diffraction grating to the 0.0 degree position. In this setting 589 nm light will shine on the photocell.
3) Turn the COLLECTOR VOLTAGE all the way to the right to apply a reverse bias voltage of at least 3 V.
4) Confirm that the dimmer plate (filter) is in place, close the light source slit completely, and turn the LAMP switch ON. (**Be careful!** On some instruments the slit opens by turning their dial to the right and closes by turning left.)

5) Using the ZERO ADJ. dial adjust the current to 0.0 $\mu$A.
6) Turn the COLLECTOR VOLTAGE dial all the way to the left to set the reverse bias potential to 0 V.
7) **Slowly** open the light source slit until the photoelectric current is 100 $\mu$A and check that the current does not change for more than a minute. When testing short wavelengths remove the dimmer plate if the photoelectric current does not reach 100 $\mu$A.
8) **Slowly** turn the reverse bias dial to 3 V and adjust the ZERO ADJ. dial so that the current reads 0.0 $\mu$A. Confirm that the current does not change for more than a minute.

   **Note** If the reverse bias dial is turned too quickly the meter cannot keep up and the true current value will not be displayed. The purpose of these tasks is to standardize the instrument for measurements at 0 and 100 $\mu$A for all wavelengths.

**Measurement**

1) While lowering the reverse bias from 3 V, make a note of when the photoelectric current begins to rise from 0.0 $\mu$A.
2) Then, from that point continue to slowly lower the reverse bias and record the corresponding photoelectric current at each voltage. Take data in steps of 0.01 V until the current reaches 0.2 $\mu$A. Above 0.2 $\mu$A, take data in increments of 0.1 V. Finally record the current when the reverse bias is 0 V.
3) Change the position of the diffraction grating in increments of 2.0 degrees and measure the relationship between reverse bias and photoelectric current in the same manner for at least the 589 nm, 539 nm, 489 nm, 437 nm, and 386 nm wavelengths. Be careful to read the dial and set it the correct wavelength. Close the light source's slit and repeat steps (7) and (8) from section (B) for each wavelength before you begin your measurements.
4) When all measurements have been completed turn all dials other than the zero-point adjustment dial completely to the left. Afterwards turn off the LAMP and POWER switches as well as the POWER switch for the ammeter and voltmeter. Disconnect all wires and return them to their designated locations.

## §6 Data Analysis

Make a table of photoelectric current and reverse bias values for each of your measured wavelengths of light.

1) Using the information in your table make a semi-log graph of photoelectric current

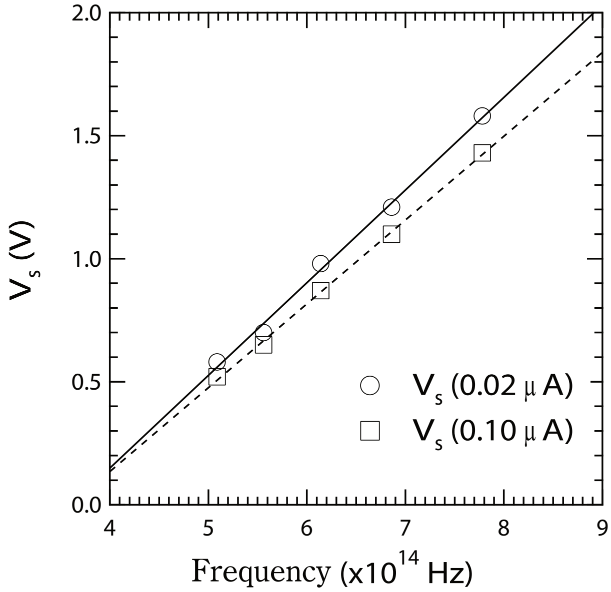



**Fig 9.6** Example of the relationship between stopping potential $V_s$ and frequency $\nu$.

versus reverse bias.

2) Take the intersection point of the reverse bias with the effective zero point of the current (either 0.02 $\mu$A or 0.1 $\mu$A in this experiment) from your graph to be the stopping potential, $V_s$. Determine the stopping potential for each measured wavelength.
3) Plot your values of $V_s$ versus their corresponding frequency $\nu$ on a graph and verify they have the same relationship as shown in Equation (9.2). Using the least-squares method, draw a straight line through the data (see Figure 9.6).
4) According to Equation (9.2) the slope of this line is $h/e$. Use this to determine Planck's constant $h$ in units of $J \cdot s$.
5) Obtain the work function for the photocathode in units of eV.

## §7 Analysis Topics for Report

1) Compare your measured value of Planck's constant with its reference value.
2) Compare your measured value of the work function with the reference value.
3) Discuss possible errors or uncertainties in your measurements and how they affect your results.
4) Search for and discuss other methods of determining Planck's constant.

## §8 Advanced Topic - the Asymptotic method

### a) Thermal excitation effects from the Fermi-Dirac distribution

If you attempt to measure the photoelectric current precisely, you will find that even if the the reverse bias is raised above $V_m$ a small photoelectric current will remain. This makes it difficult to measure $V_m$ experimentally.

Assuming the conduction electrons in a metal are at an absolute temperature $T$, the probability that they are in state with energy $E$ is given by the Fermi-Dirac distribution function,

$$f(E) = \frac{1}{1 + \exp[(E - E_{\mathrm{F}})/k_{\mathrm{B}}T]}. \tag{9.3}$$

Here $k_B$ is the Boltzmann constant and $E_f$ is called the Fermi Energy. As shown in panel (a) of Figure 9.7 at absolute zero ($T = 0$) the Fermi-Dirac distribution is a step function with $f(E) = 1$ when $E < E_f$ and is zero otherwise. All of the conduction electrons have energy less than the Fermi energy. If a conduction electron then receives energy $h\nu$ from a photon it will use energy equal to the metal's work function to escape the metal and become a photoelectron with kinetic energy, $K$. A photoelectron electron with the maximum kinetic energy, $K_{max} = h\nu - W$ is a conduction electron whose initial energy was $E_f$.

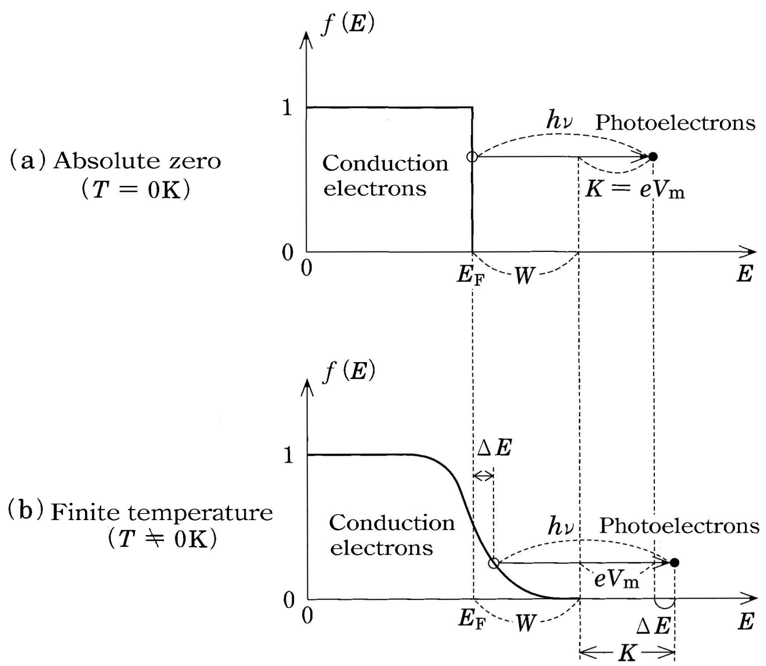



**Fig 9.7** Fermi-Dirac $f(E)$ distribution of conduction electrons in a metal and the photon-induced emission of a conduction electron.

If a reverse bias is applied so as to suppress the maximum photoelectron energy $K_{max}$, $V = V_m(eV_m = K_{max} = h\nu - W)$ then no photoelectrons can reach the collector and the photoelectric current drops to zero. However, if the metal is at a finite temperature ($T > 0$) then electrons near the Fermi energy carry thermal energy and as a result some will have $\Delta E$ more energy than $E_f$. In this case the distribution function becomes smooth around $E = E_f$ as shown in panel (b) of the figure. Accordingly, electrons with energy $E_f + \Delta E$ which receive energy $h\nu$ from photon still have kinetic energy $\Delta E$ even under a reverse bias of $V_m$. These electrons will therefore reach the collector and be observed as a current.

Using this type of model, the photoelectric current $I_{photo}$ emitted perpendicular to the metal surface and arriving at the collector under a reverse bias $V$ is given by the following equation.

$$I_{photo} = AT^2F(x), \tag{9.4}$$

where $A$ is a quantity that depends on the frequency $\nu$, and potential $V_m$ where

$$x = \frac{e(V_m - V)}{k_B T}, \tag{9.5}$$

$$eV_m = h\nu - W, \text{ and} \tag{9.6}$$

$$F(x) = \int_0^\infty \ln\{1 + \exp(x - it)\}dt. \tag{9.7}$$

When $T = 0$ the quantity $eV_m$ corresponds to the maximum energy of a photoelectron.

Figure 9.8 shows the relationship between the photoelectric current $I_{photo}$ and the reverse bias $V$ as given by Equation (9.4). As can be seen in panel (a) of the figure, for sufficiently low photocathode temperatures the current exists only up to $V_m$. As the temperature increases current is observed even above $V_m$ and there is no longer a clear maximum energy. Panel (b) of the figure shows how $I_{photo}$ depends on $V$ for various values of $V_m$. Note that increases in $V_m$ correspond to a translation along the $V$ axis. This can also be seen in Equation (9.4). Essentially, on a semi-logarithmic graph changing $V_m$ causes a shift in the horizontal axis by $F(x)$ and a shift in the vertical axis by $\log A$.

**b) Correcting for thermal excitation using the asymptotic method**

When the reverse bias is sufficiently large and $e|V_m - V| \gg k_B T$ then Equation (9.4) becomes $I_{photo} \approx \exp[e(V_m - V)/k_B T]$ and $\log I_{photo}$ is proportional to $V$. If the photocathode temperature is not too high, the voltage at which the curve begins to separate from the asymptote approximates the maximum electron energy $eV_m$ when $T = 0K$, as shown in Figure 9.8. In the following you will determine the maximum voltage $V_m$ for a variety of frequencies $\nu$ and using the relation

$$eV_m = h\nu - W \tag{9.8}$$

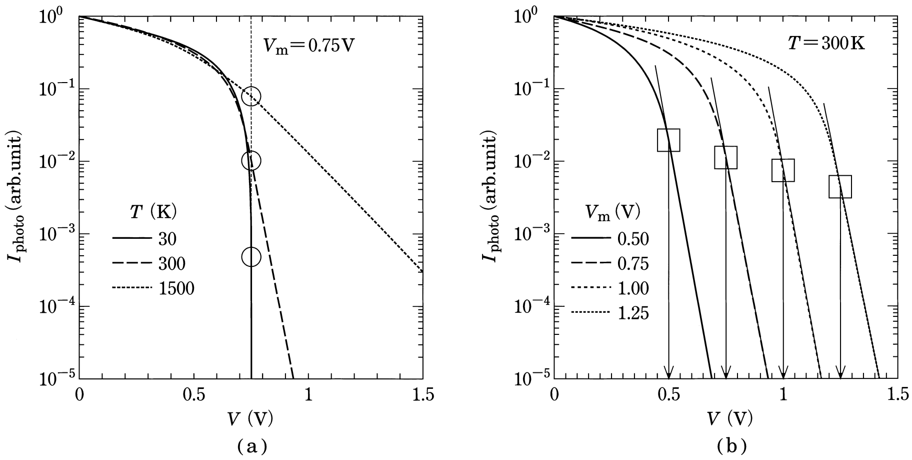



**Fig 9.8** Relationship between photoelectric current and reverse bias as predicted in Equation (9.4) shown on a semi-logarithmic plot. The photoelectric current $I_{photo}$ is shown on a logarithmic vertical axis and though it decreases suddenly with larger reverse bias $V$, it never reaches zero. Panel (a) shows the temperature dependence. As the temperature increases, even if $V$ exceeds $V_m$ photoelectric current still flows. Panel (b) shows the dependence on $V_m$ for $T$ fixed to $300K$. The curves have been normalized so that the photoelectric current is 1 when $V = 0$. Changing $V_m$ is experimentally equivalent to changing the light frequency $\nu$. Notice that in both panels the value of $V$ where the curves begins to deviate from the asymptote is roughly $V_m$. In the figures these points are marked by the circles and squares.

you will determine Planck's constant $h$.

**c) Data analysis using the asymptotic method**

(1) Draw a straight line asymptote over the data corresponding to the longest wavelength data in your experiment. Draw asymptotes with the same slope over each of your other data sets. These asymptotes should pass through as many data points as possible, but the number will decrease for data taken at shorter wavelengths. (See Figure 9.8(b))

(2) Obtain the maximum electron energy $eV_m$ at $T = 0$ for each data set by assuming the voltage where the data begins to deviate from the asymptote corresponds to the maximum electron energy.

(3) Using the obtained values of $V_m$ make a plot similar to that from the stopping potential method of Figure 9.6. Plot the values on the vertical axis with their corresponding frequency $\nu$ on the horizontal axis. Verify the data obey the relationship predicted in Equation (9.8) and draw a smooth curve through the points.

(4) Determine the value of Planck's constants in units of $J \cdot s$ using the slope of the curve.

(5) Obtain the work function for the photocathode in units of eV.

# 1. Appendix

## §1 Probability and Statistics

When we toss a coin into the air and catch the falling coin by hand, the probability that it comes on the palm with heads up and the probability for the coin to come with tails up are ideally $\frac{1}{2}$ for each. If we repeat this procedure of tossing the coin (coin-toss) again and again, and increase the number of trial to large enough, then the probability for heads up and that for the tails up approach $\frac{1}{2}$ asymptotically. After $m$ trials are performed the probability for heads (tails) up for $n$ times becomes a certain distribution as discussed in the following.

**a) Binomial distribution** When we throw a dice, the probability to find any one of six faces up is ideally equal to $\frac{1}{6}$. After $m$ trials were done, the number of times, $n$, to have the face '1 'up is given by the following probability distribution. Namely, if the probability for a event to happen for one trial is $p$, then the probability for this event to occur for $n$ times within the total $m$ independent trials, is expressed as the expression below. This probability distribution is called **binomial distribution**.

$$P(n) = \binom{m}{n} p^n(1-p)^{m-n} = \frac{m!}{(m-n)!n!}p^n(1-p)^{m-n}. \tag{1.1}$$

This can be explained as follows. For an event, with a probability $p$, to occur for $n$ times gives rise to a factor $p^n$ , and for the event not to occur for the remaining $m-n$ times gives another factor $(1-p)^{m-n}$. The number of ways for such situation is given by the combination $\binom{m}{n}$, which is the third factor. The product of these three factors leads to the above probability distribution.

Now the probability distribution $P(n)$ satisfies the following properties.

$$\sum_{n=0}^{m} P(n) = 1 \qquad \overline{n} \equiv \sum_{n=0}^{m} nP(n) = mp \ . \tag{1.2}$$

Thus the mean value (average value) for this event to occur is $\overline{n} = mp$. While, the standard deviation $\sigma$ is defined by the square root of the following quantity:

$$\sigma^2 = \overline{(n-\overline{n})^2} = \sum_{n=0}^{m}(n^2 - 2n\overline{n} + \overline{n}^2)P(n) = \overline{n^2} - \overline{n}^2 \ . \tag{1.3}$$

For the case of binomial distribution, $\overline{n^2}$ is found to be

$$\begin{aligned}\overline{n^2} &= \sum_{n=0}^{m} n^2 P(n) = \sum_{n=1}^{m} \frac{nm!}{(m-n)!(n-1)!} p^n (1-p)^{m-n} \\ &= \left[\sum_{n=2}^{m} \frac{m!}{(m-n)!(n-2)!} + \sum_{n=1}^{m} \frac{m!}{(m-n)!(n-1)!}\right] p^n (1-p)^{m-n} \\ &= m(m-1)p^2 + mp \end{aligned} \tag{1.4}$$

Therefore the square of the standard deviation turns out to be

$$\sigma^2 = m(m-1)p^2 - 2mp \cdot mp + (mp)^2 = mp(1-p) \tag{1.5}$$

So we finally get $\sigma = \sqrt{mp(1-p)}$.

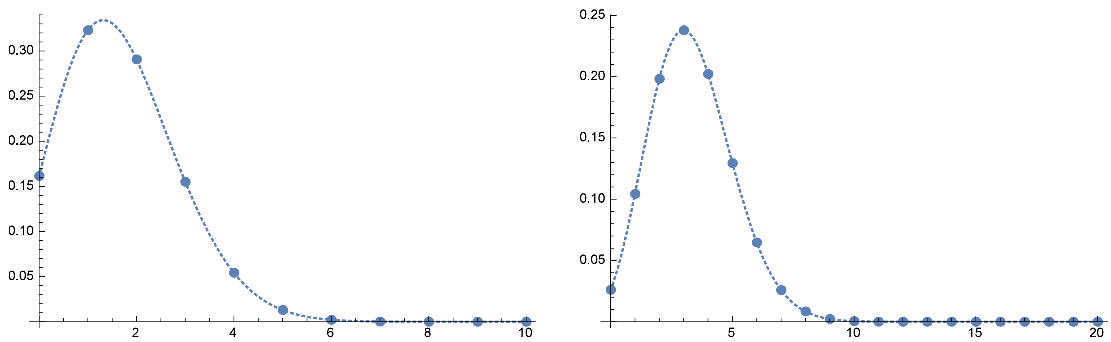



**Fig 1.1** (a) binomial distribution $p = 1/6$ $m = 10$ (b) binomial distribution $p = 1/6$ $m = 20$

We have shown in Figure 1.1, the binomial distribution for the case (a) $p = 1/6$ and $m = 10$ $(\overline{n} = 10/6 \approx 1.67)$ , (b) $m = 20$ $(\overline{n} = 20/6 \approx 3.33)$ . If we take the number of trials $m$ to large enough, the probability $n/m$ for the event to occur $n$ times out of the total $m$ trials approaches the constant $p$. This law is called **law of large numbers**.

**b) Poisson distribution** If we rewrite the mean value $\overline{n} = mp$ as $\lambda$ we get

$$P(n) = \frac{m!}{(m-n)!\, n!} \left(\frac{\lambda}{m}\right)^n \left(1 - \frac{\lambda}{m}\right)^{m-n} . \tag{1.6}$$

and take a limit $p \ll 1$ i.e. $n \ll m$ we find

$$\begin{aligned} P(n) &= \frac{m(m-1)\cdots(m-n+1)}{n!} \left(\frac{\lambda}{m}\right)^n \left(1 - \frac{\lambda}{m}\right)^{m-n} \\ &\to \frac{\lambda^n}{n!}\left(1 - \frac{\lambda}{m}\right)^m \approx \frac{\lambda^n}{n!} e^{-\lambda} \end{aligned} \tag{1.7}$$

This is called **Poisson distribution**. We have shown the case $\lambda = 3$ in Figure 1.2.

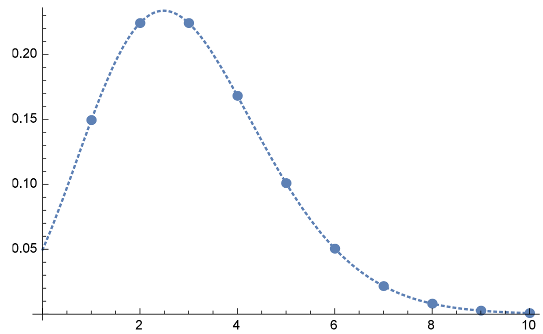



**Fig 1.2** Poisson distribution $\lambda = 3$

The characteristic of this distribution is that the mean value $\overline{n}$ is equal to the square of the standard deviation $\sigma^2$ and is given by $\lambda$ as follows:

$$\overline{n} = \sum_{n=1}^{\infty} nP(n) = \lambda, \quad \sigma^2 = \sum_{n=1}^{\infty} (n - \overline{n})^2 P(n) = \overline{n^2} - \overline{n}^2 = \lambda . \tag{1.8}$$

**c) Normal distribution** As we have seen, for the binomial distribution:

$$P(n) = \binom{m}{n} p^n (1-p)^{m-n} = \frac{m!}{(m-n)!n!} p^n q^{m-n} , \quad q = 1 - p \tag{1.9}$$

the mean value is $\overline{n} = mp$ and the standard deviation is $\sigma = \sqrt{mpq}$.

Now let us intoeduce a new variable $t$ defined as

$$t = \frac{n - \overline{n}}{\sigma} = \frac{n - mp}{\sqrt{mpq}} \tag{1.10}$$

and then for the variation of $n$ with $0, 1, \cdots, m$, the value $t$ has an equally spaced interval $1/\sqrt{mpq}$, and the probability can be rewritten in terms of the new variable $t$ as

$$P(t) = \frac{m!}{n!(m-n)!} p^n q^{m-n} \tag{1.11}$$

In the limit of $m$ going to infinity, we can apply the Stirling's formula for $m!$, $n!$, $(m-n)!$ and get

$$\log\left(\sqrt{2\pi} P(t) / \frac{1}{mpq}\right) = -\frac{t^2}{2} + \mathcal{O}(1/\sqrt{m}) \tag{1.12}$$

Namely, we obtain

$$P(t)/\frac{1}{mpq} \sim \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \tag{1.13}$$

and realize that the limit of the binomial distribution becomes **normal distribution** as shown in Figure 1.3.

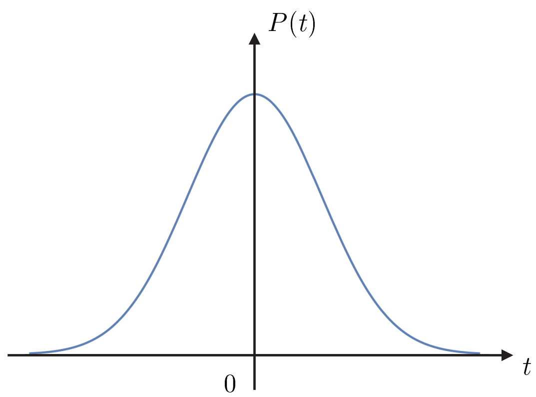



**Fig 1.3** Normal distribution

For the distribution of measured value in the large number of measurement we have the law of error distribution obtained by Gauss. Now the error distribution for the measurement of a certain quantity is given by the following normal distribution (Gaussian distribution) with mean value 0.

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \quad x = X - X_0, \quad X_0 : \text{true value}, \quad X : \text{measured value}$$

(1.14)

## §2 Method of Maximum Likelihood

Let us suppose that we have measured a physical quantity $X$ for $n$ times and obtained the data $X_i$ $(i = 1, 2, \cdots, n)$. We usually adopt the mean value $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ as the experimental data. In the following we will show that this choice is equal to the most probable or estimted value $X_0$ from the method of maximum likelihood method.

We now assume that the distributuion of meassured values is given by normal distribution (Gaussian distribution) with the central value (most probable value) $X_0$ and the stadard deviation $\sigma$. For the case of $i$-th measurement, the probability for the measured value to take some value between $X_i$ and $X_i + dX_i$ is given as

$$dP_i = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(X_i - X_0)^2}{2\sigma^2}\right] dX_i \tag{1.15}$$

The difference between the most probable value $X_0$ and the measure value $X_i$:

$$X_i - X_0 \equiv x_i \tag{1.16}$$

is called **residual error**. Thus $dP_i$ can be also wriien as

$$dP_i = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{x_i^2}{2\sigma^2}\right] dx_i \equiv P(x_i)dx_i \tag{1.17}$$

The probability for us to get the residual errors, $x_1,\ x_2,\ \cdots,\ x_n$, after performing a set of measurements. reads

$$P = \prod_{i=1}^{n} P(x_i)dx_i = (\frac{1}{\sqrt{2\pi}\sigma})^n \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{n} x_i^2\right] dx_1 \cdots dx_n \tag{1.18}$$

The condition for $X_0$ to be the most probable value is nothing but the condition for $P$ to become maximum. This means that the sum of the squared residual errors:

$$S = \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} (X_i - X_0)^2 \tag{1.19}$$

should be minimum. To this end the condition for $X_0$ turns out to be

$$\frac{\partial S}{\partial X_0} = 0 \tag{1.20}$$

that is

$$2\left(\sum_{i=1}^{n} X_i - nX_0\right) = 0 \qquad \text{すなわち} \quad X_0 = \frac{1}{n}\sum_{i=1}^{n} X_i\ . \tag{1.21}$$

Thus it has been proved that the most probable value $X_0$ is equal to the mean value $\bar{X}$.

## §3 Method of Least Squares

Let us consider the case where we measure a physical quantity $Y$ which is represented as a function of another physical quantity $X$ as $Y = f(X)$. Namely $Y$ can be calculated as a function of $X$ involving a certain number of parameters. We denote the measured values of $X$ by $x_i$ $(i = 1, \cdots, n)$ and the measured values of $Y$ by $y_i$ with errors $\sigma_i$ $(i = 1, \cdots, n)$ . In this situation the method to determine the most probable values for the parameters is so called **method of least squares**.

As an example let us consider the case where $y$ is a linear function of $x$:

$$y = f(x) = a + bx \tag{1.22}$$

where $a$ and $b$ are the parameters. We now take the following normal distribution:

$$P(a,b) = \prod_i P_i = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left\{-\frac{1}{2}\sum_i \left[\frac{y_i - f(x_i)}{\sigma_i}\right]^2\right\} \tag{1.23}$$

What we have to do is to maximize the probability distribution $P(a, b)$ to determine the parameters $a$, $b$. This means that the following square sum of the exponent of the exponetial function should be minimized.

$$\chi^2 \equiv \sum_{i=1}^{n} \left[\frac{1}{\sigma_i^2}(y_i - a - bx_i)^2\right] \tag{1.24}$$

In a simple case in which $\sigma_i$ = constant $\equiv \sigma$, the following quantity

$$S \equiv \sum_{i=1}^{n} (y_i - a - bx_i)^2 \tag{1.25}$$

should be minimized. Therefore we set the partial derivatives of $S$ with respect to the parameters $a$, $b$ to vanish

$$\frac{\partial S}{\partial a} = -2\sum_{i=1}^{n}(y_i - a - bx_i) = 0 \tag{1.26}$$

$$\frac{\partial S}{\partial b} = -2\sum_{i=1}^{n} x_i(y_i - a - bx_i) = 0 \tag{1.27}$$

Solving the above coupled equations, we obtain $a$ and $b$ as follows:

$$a = \frac{1}{\Delta}\left(\sum x_i^2 \cdot \sum y_i - \sum x_i \cdot \sum x_i y_i\right) \tag{1.28}$$

$$b = \frac{1}{\Delta}\left(n\sum x_i y_i - \sum x_i \cdot \sum y_i\right) . \tag{1.29}$$

where $\Delta$ is given by

$$\Delta = n\sum x_i^2 - \left(\sum x_i\right)^2 \tag{1.30}$$

and the summation over the suffix $i$ from 1 to $n$ is tacitly understood without explicitly mentioning it and we omit the suffix. The method to determine the functional form through the above procedure is called **method of least squares**.

Now we consider the errors of $a, b$, *i.e.* $\sigma_a, \sigma_b$. In order to apply the law of error propagation, we partially differentiate the Equation (1.28) with respect to $y_j$ $(i = 1, \cdots, n)$

$$\frac{\partial a}{\partial y_j} = \frac{1}{\Delta}\left(\sum x_i^2 - x_j \sum x_i\right) \tag{1.31}$$

hence we find $\sigma_a$ as

$$\sigma_a = \sqrt{\frac{\sigma^2}{\Delta^2}\left(\sum x_i^2 - x_1 \sum x_i\right)^2 + \cdots + \frac{\sigma^2}{\Delta^2}\left(\sum x_i^2 - x_n \sum x_i\right)^2} \tag{1.32}$$

$$= \frac{\sigma}{\Delta}\sqrt{n\left(\sum x_i^2\right)^2 - 2\sum x_i^2 \cdot \left(\sum x_i\right)^2 + \sum x_i^2 \cdot \left(\sum x_i\right)^2} \tag{1.33}$$

$$= \frac{\sigma}{\Delta}\sqrt{\sum x_i^2} \cdot \sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2} \tag{1.34}$$

$$\sigma_a = \sqrt{\frac{1}{\Delta}\sum x_i^2} \cdot \sigma \ . \tag{1.35}$$

Similarly by differentiating Eq.(1.29) we get $\sigma_b$ as

$$\frac{\partial b}{\partial y_j} = \frac{1}{\Delta}\left(n x_j - \sum x_i\right) \tag{1.36}$$

$$\sigma_b = \sqrt{\frac{1}{\Delta^2}\left(n x_1 - \sum x_i\right)^2 \sigma^2 + \cdots + \frac{1}{\Delta^2}\left(n x_n - \sum x_i\right)^2 \sigma^2} \tag{1.37}$$

$$\sigma_b = \sqrt{\frac{n}{\Delta}} \cdot \sigma \tag{1.38}$$

Now we estimate the error $\sigma$. The data $y_i$ $(i = 1, \cdots, n)$ distributes around $ax + b$ in the form of normal distribution. Since we determine two unknown parameters $a, b$ by using the $n$ data, the number of the independent data now becomes $n - 2$. Taking account of this fact, we find

$$\sigma \sim \sqrt{\frac{1}{n-2}\sum (y_i - a - bx_i)^2} = \sqrt{\frac{S}{n-2}} \tag{1.39}$$

In the case $\sigma_i$ $(i = 1, \cdots, n)$ are not constant, we minimize the $\chi^2$ itself. That is

$$\frac{\partial \chi^2}{\partial a} = -2\sum_{i=1}^{n}\left[\frac{1}{\sigma_i^2}(y_i - a - bx_i)\right] = 0 \tag{1.40}$$

$$\frac{\partial \chi^2}{\partial b} = -2\sum_{i=1}^{n}\left[\frac{1}{\sigma_i^2}x_i(y_i - a - bx_i)\right] = 0 \tag{1.41}$$

Solving the above coupled equations for $a$, $b$, we have

$$a = \frac{1}{\Delta}\left(\sum \frac{x_i^2}{\sigma_i^2}\cdot\sum\frac{y_i}{\sigma_i^2} - \sum\frac{x_i}{\sigma_i^2}\cdot\sum\frac{x_i y_i}{\sigma_i^2}\right) \tag{1.42}$$

$$b = \frac{1}{\Delta}\left(\sum \frac{1}{\sigma_i^2}\cdot\sum\frac{x_i y_i}{\sigma_i^2} - \sum\frac{x_i}{\sigma_i^2}\cdot\sum\frac{y_i}{\sigma_i^2}\right) \tag{1.43}$$

where $\Delta$ is given by

$$\Delta = \sum\frac{1}{\sigma_i^2}\cdot\sum\frac{x_i^2}{\sigma_i^2} - \left(\sum\frac{x_i}{\sigma_i^2}\right)^2 \tag{1.44}$$

The method to determine the parameters by using the weighted least square fit just described above is called $\chi^2$ **fit**.

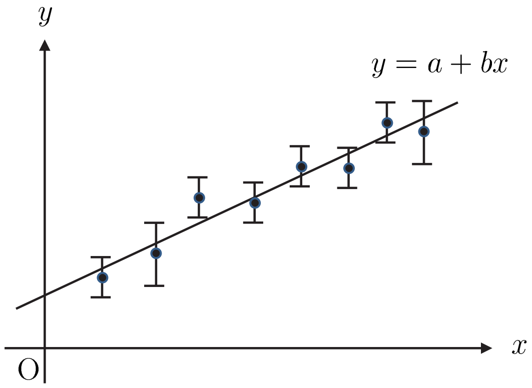



**Fig 1.4** $\chi^2$ fit

In a similar way like Eqs. (1.35), (1.38) we can calculate the errors $\sigma_a, \sigma_b$. Partially differentiating Eqs. (1.42), (1.43) with respect to $y_j$ $(i = 1, \cdots, n)$, we get

$$\frac{\partial a}{\partial y_j} = \frac{1}{\Delta}\left(\frac{1}{\sigma_j^2}\sum\frac{x_i^2}{\sigma_i^2} - \frac{x_j}{\sigma_j^2}\sum\frac{x_i}{\sigma_i^2}\right) \tag{1.45}$$

$$\frac{\partial b}{\partial y_j} = \frac{1}{\Delta}\left(\frac{x_j}{\sigma_j^2}\sum\frac{1}{\sigma_i^2} - \frac{1}{\sigma_j^2}\sum\frac{x_i}{\sigma_i^2}\right) \tag{1.46}$$

Hence $\sigma_a, \sigma_b$ turn out to be

$$\sigma_a = \sqrt{\frac{1}{\Delta}\sum\frac{x_i^2}{\sigma_i^2}} \qquad \sigma_b = \sqrt{\frac{1}{\Delta}\sum\frac{1}{\sigma_i^2}}\ . \tag{1.47}$$

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**Physics Experiments**

2016 年 9 月　第 1 版　第 1 刷　発行
2017 年 9 月　第 2 版　第 1 刷　発行
2018 年 9 月　第 3 版　第 1 刷　発行

編　者　京都大学大学院人間・環境研究科 物質相関論講座
京都大学国際高等教育院 物理学部会
発行者　発 田 和 子
発行所　株式会社 学術図書出版社
〒113−0033　東京都文京区本郷 5 丁目 4 の 6
TEL 03−3811−0889　振替　00110−4−28454
印刷　三和印刷 (株)

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ISBN978−4−7806−0675−1　C3042

Graph Paper for Experiment 9 “Measurement of Planck’s Constant” ⇨